Statistical Analysis of Individual Solar Active Regions

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“How can there be evolution if things are in balance? Systems in balance or equilibrium, by definition, do not go anywhere.”

Per Bak
Parker’s nanoflare heating theory requires flare frequency distributions with powerlaw slopes greater than 2. Oppositely, the Lu&Hamilton Cellular Automaton predicts such slopes to be less than 2 and statistically indistinguishable for different active regions. To investigate these claims, I identify individual active regions that acted mostly alone on the Sun. Their flare lists are constructed and their x-ray timeseries are analyzed. Flare and background coronal radiation are separated. Flare thermal energy timeseries are created. The flare events, both in x-rays and thermal energy, are statistically analyzed with cumulative distributions and scatter plots. The flare and background timeseries are analyzed with fourier plots and correlation functions. Half of the regions studied show a single, less than 2 powerlaw in energy distributions, and qualitative agreement with the LH model. The other half show two slopes, a less than 2 from lowest to mid energies, and a greater than 2 from mid to highest energies. Thus I propose that not all ARs can reach a true SOC state. Possible causes could be the presence of strong drivers, inability to reach SOC configurations uniformly, or nonconservative next-neighbor interactions. Flares are found to be independent events in time. The driver of active regions is time-varying, alternating between a slow and a fast average rate. Background coronal radiation is independent of flaring activity, shows long correlation times and powerlaw fourier spectra, and could thus be a slowly driven SOC system.
Acknowledgements

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A The Analysis Algorithm in IDL
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To my family,
for their love,
trust
and support
Chapter 1

Introduction

1.1 The Coronal Heating Problem

Our nearest star, the Sun, is a huge sphere of plasma. It is a typical G-type, middle-aged, main sequence star born some 4.6 billion years ago. What makes it unique for us Earthliers is first and foremost the fact that the energy it radiates has created life as we know it. But for astrophysicists, the Sun is even more important, as it is our only chance to study a star at such close proximity. Dedicated solar observations of sunspots starting from the mid 1600’s have revealed the 9-12 year solar cycle, during which the levels of activity of the Sun oscillate from a maximum to a minimum.

The advent of the space age made possible the measurement of solar radiation across the whole electromagnetic spectrum. Through such observations we have realized that the Sun is in fact a variable star. Explosions of power equal to billion atomic bombs, known as solar flares, happen in the solar atmosphere daily. Solar material bursts out of the solar atmosphere and expands into space, known as coronal mass ejections. Particles are accelerated in the solar atmosphere and stream away from the Sun, known as solar energetic particles.

During a total solar eclipse in 1896, Grotrian and Edlen turned their spectrophotometers towards the solar corona and observed some sets of emission lines that shouldn’t have been there. They didn’t belong in the spectrum of Hydrogen, of Helium or of any other element previously known to exist on the Sun. In the beginning scientists believed they had discovered a new element on the Sun. Subsequently they realized these lines are part of a well-known element’s spectrum, the Iron spectrum. Calculations indicate that at temperatures of $10^6$ Kelvin and above an iron gas emits photons that match the Grotrian and Edlen lines. A problem arose due to this interpretation though. How could it be that the atmosphere of the Sun is that hot when the whole spectrum of solar radiation is a well defined black body at a temperature of 6000 Kelvin?

Astrophysicists and nuclear physicists had already agreed back then that the only possible energy generator in a main sequence star lies in its core. It is the process of nuclear fusion of Hydrogen to Helium. The energy created in the core is transported outwards through the various solar layers and parts of it are deposited in the material along the way. It should be then that the temperature drops as one moves away from the core of the Sun to its atmosphere. But the solar corona has to be at $10^6$ degrees Kelvin, exceeding the temperature of the photosphere. Using simple logic, this observation is against the First Law of Thermodynamics which requires that heat flows from hotter to cooler regions.
1.2 The Solution: Solar Magnetism

It was soon realized that another crucial observation has the potential to explain the paradox: the Sun has a magnetic field. Just like the Earth, the Sun is generating its own magnetic field, constructed in the convection zone below the photosphere. According to the solar dynamo theory, the steep temperature gradient between the bottom and top of the convection zone sets the plasma in upstream/downstream boiling motions. These currents are also swirling to follow the solar rotation. According to Ampère’s law such currents generate magnetic fields which, due to the variability of their corresponding currents, are themselves variable. According to Faraday’s law variable magnetic fields induce electric fields and thus new currents, creating a self-maintained, closed feedback process.

The magnetic field is concentrated into bundles, known as flux tubes, and it is buoyant. Through processes not well understood, it rises from the convection zone to the photosphere, penetrating it, and filling the solar atmosphere up to coronal heights. Although only large tubes in solar active regions are observed, it is theorized that the whole corona is comprised of a huge number of small, thin flux tubes. These are invisible because they don’t produce significant radiation.

Why is the presence of magnetic fields in the Sun able to explain the high coronal temperatures? Because the magnetic field is the only linking agent between the convection zone/photosphere, where energy is abundant, and the corona, where no substantial energy source exists. Furthermore, magnetic fields have a unique characteristic: they store energy in them due to impedance effects.

The magnetic field lines behave very much like elastic rubber bands. They are stressed along their whole length when tension is applied to some part of them and accumulate energy due to stress. The parts of the lines in the convection zone and photosphere are constantly stressed due to variable plasma motions. In electromagnetic terms, the changes in currents affect all parts of the magnetic lines, which store energy resisting the change. It is believed that the turbulent nature of the plasma motions is mapped onto the topology of stresses along the lines, creating fractal structures of stresses. The dissipation of energy from the destressing of the lines is enough to sustain a corona that hot.

The accumulated stresses are mostly released inside the corona, as opposed to the chromosphere, because there the magnetic flux tubes overlap significantly. This overlap is necessary because magnetic field lines need to be in close proximity in order to interact and connect into larger connectivity clusters. These clusters are called unstable reconnection current sheets (UCS) (‘unstable’ for not long lived). It is assumed that a single tube would need to acquire great amounts of stresses in order to "exceed the break limit" at a certain place along its length (meaning the onset of various types of magnetic instabilities) and release energy. On the other hand, the accumulated stresses in UCSs exceed the "break limit" easier and faster and thus offer their energy more efficiently to the coronal plasma. The coronal magnetic topology, comprised of self-similar, all size and shape UCSs, is continuously driven by the convection zone/photospheric motions that add new stresses into the system. All the above are formally known as magnetic reconnection, a process through which the magnetic topology responds to change by rearranging itself and releasing energy to restabilize itself.
1.3 Heat Release and Transport in the Corona

During reconnection, the accumulated stresses of a UCS are redistributed among neighboring regions and thus relaxed. As a result, new UCSs are formed elsewhere in the topology. Some part of the excess stress is not redistributed but converted into free energy, and given to the plasma inside and around the sheet. Just like some energy is lost into heat in any energy conversion/redistribution process in nature, in reconnection too some energy is lost into the plasma. The coronal plasma processes this extra energy mainly by raising its temperature, thermalizing the Maxwellian population of plasma in the reconnection site.

The plasma in the corona is low-$\beta$, meaning that magnetic pressure and not thermal pressure determines the dynamic response. As a result, the motion of plasma particles is controlled by magnetic pressures along the flux tubes, restricting the omnidirectional and expansion movements that Maxwellian particles normally want to follow. This is formally known in the literature as the “force-free condition” of the corona. The inability of particles to have velocity components perpendicular to the magnetic field results in the lack of Lorentz forces on plasma particles. Thus currents are mainly flowing along and not across flux tubes, making the path of the tubes highly conductive.

Due to the force-free condition, the released heat of a reconnection event will efficiently be transported along and not across flux tubes. The thermalization is in fact split into two different thermalization subprocesses:

- first, in situ heating of plasma inside and around the UCS (direct heating)
- then, heating of plasma along magnetic field lines (heat conduction)

Both thermalization subprocesses are assumed to be happening through particle collisions in free-free (ion-electron) and free-bound (heavy nuclei-electron) Brehmsstrahlung processes.

Numerical MHD simulations give insight into the process of in situ heating. Hansteen et al. (2015) find that although on large scales heating occurs along loop-shaped structures, on small scales heating is concentrated in current sheets. Toutountzi et al. (2016) find that the presence of collisions trap the bulk of the plasma around UCSs. Heating happens within tens of seconds for electrons and thousands of seconds for ions. The temperatures in this bulk plasma trapped around UCs are in excess of 2 MK.

1.4 Nanoflare Heating

From the above, a natural question arises: If heat conduction is only efficient along flux tubes, how can the whole corona be heated? The answer is the Parker nanoflare scenario. Parker envisioned a corona full of tiny magnetic flux tubes, a scaled down version of the picture we observe in active regions, repeated all over the corona. Small reconnection events must be happening at a high occurrence rate along each flux tube, heating them individually. The great number of tubes guarantees that all parts of the corona are heated equally, even though cross-conduction is prohibited. These small events must be scaled down versions (in time and spatial extent) of the large events we observe in active regions, the solar flares. This is why we call them nanoflares. In contrast to flares, nanoflares must be happening all the time in the unresolved corona, in order to supply it with sufficient amounts of energy and maintain it at $10^6 K$. When Parker formulated his
scenario, instruments were not sensitive enough to resolve nanoflares neither in space nor in time. Today, the observation of nanoflares is still a much anticipated scientific discovery, yet nanoflares are evident in simulations and can be deduced indirectly from observed data based on their expected signature.

1.5 Soft X-ray Production

In this study soft x-ray timeseries are analyzed and used as an tool to investigate coronal heating. How are soft x-rays created in the corona? Apart from conducting the added energy in the plasma, collisions also release part of the energy as radiative losses. This is why the corona is so 'bright' along flux tubes at high energy wavelengths such as EUV and X-rays. This radiation is the tale-telling signature that solar physicists use in order to understand coronal heating.

Studies of 1D hydrodynamic models (Cargill et al. (2004), Reale (2007)) simulate the process of heating and radiation in loops, termed chromospheric evaporation. Essentially this is the same process for events of all sizes, from nanoflares to microflares to large flares, and what only changes is the spatial and the time scale of the events. As discussed in Parnell et al. (2012), an impulsive coronal heating event will cause the strand to evolve through four phases:

1. First (short) phase: Heat from the event is conducted along the field, rapidly raising the temperature of the whole strand up to a maximum. The density and emission show very little change during this very short phase, creating a strand that is underdense relative to static equilibrium theory.

2. Second (longer) phase: The impulsive event continues to heat, the temperature remains constant, but the chromosphere is strongly heated. As a result, it expands and evaporates into the coronal part of the strand. The emission rises rapidly and the strand becomes denser.

3. Third phase: Starts when the impulsive event stops and conduction starts to cool the plasma. The emission first peaks, then starts to decrease, but the density continues to increase. As the strand cools and becomes more dense, radiative cooling effects become more important. At the end of this phase radiative and conductive cooling balance.

4. Fourth phase: The loop appears overdense, radiative cooling dominates. The temperature, density and emission measure start to decrease until they reach their pre-event values.

As discussed in Klimchuk et al. (2006), the overall effect of high occurrence nanoflares must be a steady level of soft x-radiation, which could explain the background we see in typical timeseries from the Sun. Larger (A to X-class) flares must be the result of reconnections in large UCSs. Due to their large size and low occurrence rate, they can be individually resolved and appear as impulsive radiation that ‘sits’ on top of the background.

1.6 Statistics of Solar X-ray Flares

1.6.1 Powerlaws

It is widely accepted nowadays, based on observational studies, analytical models and computer simulations, that the solar x-ray flare distribution follows a
1.6. Statistics of Solar X-ray Flares

Powerlaw form:

\[ N \sim S^{-\alpha} \]  

where \( N \) is the number of observed events in a range of sizes around \( S \), \( S \) is a measure of event size, and \( \alpha \) is the characteristic exponent, the quantity we wish to calculate from flare observations.

Powerlaw distributions are among the most important distributions for many scientific disciplines. As Bak (1996) describes in his book, they have the following properties:

1. when plotted on log-log axis their fits are lines extending over many scales. This means that the same mechanism works on all scales. The largest events of a powerlaw distribution do not play a special role; they follow the same law as the smallest events. Consequently, a general theory to explain the powerlaw form must encompass all events, small and large.

2. powerlaw distributions are scale-free and self-similar, meaning there is no typical size for the events they describe, no well defined mean. This can clearly be understood if one thinks of their linear form in log-log axis. No spike exists in the distribution as is the case for example of the Maxwellian distribution of speeds that has a well-defined average value.

3. powerlaws are universal in that they appear to describe the distributions from very diverse phenomena.

All the above imply that we can work with whatever part of the x-ray flare distribution is observationally available (due to instrumental limitations), but still deduce information about the unobserved parts of it. In this study, we are analyzing the statistics of large flares, but in an effort to gain insight into the statistics of nanoflares.

The determining factor of a powerlaw distribution is the exponent \( \alpha \). In order to calculate its value, distributions are constructed from observations of x-ray flare events. \( S \) is some measure of a flare’s magnitude, such as its peak count rate, its total count rate, its peak flux, its fluence, as they are measured in some wavelength range, or its peak thermal/nonthermal energy and total thermal/nonthermal energy as derived from measurements. Two types of distributions can be constructed:

- If the dataset is medium or large, frequency distributions \( f(S) \) (hereafter referred to as freq.dist.) or number distributions (hereafter referred to as num.dist.) \( N(S) \) are constructed.
- If the dataset is small, cumulative distributions \( F(S) \) (hereafter referred to as cum.dist.) or cumulative number distributions (hereafter referred to as cum.num.dist.) \( C(S) \) are constructed.

1.6.2 Parker’s Conjecture

Let \( f(E)dE \) be the fraction of flares releasing an amount of energy between \( E \) and \( E + dE \) per unit time. As seen from solar flare observations:

\[ f(E) = f_{0}E^{-\alpha}, \quad \alpha > 0 \]  

(1.2)
Then the total energy released per unit time by the ensemble of flares is:

\[ E_{\text{tot}} = \int_{E_{\text{min}}}^{E_{\text{max}}} f(E) E dE = f_o \left( \frac{E^{2-\alpha}}{2-\alpha} \right) E_{\text{max}}^{\alpha-\alpha} \], \quad \alpha \neq 2 \quad (1.3)

- If \( \alpha < 2 \): the largest flares of the distribution dominate the release of energy. These largest flares are the source of energy for the corona.
- If \( \alpha > 2 \): the smallest flares are energetically dominant. These smallest flares are the source of energy for the corona.

Up to today, no observational freq. dist. have been able to find a \( >2 \) slope, given the instrumental capabilities of our instruments. Thus Parker’s conjecture remains an open question.

### 1.6.3 Soft X-ray Observations

Flare observations are divided into two categories: the soft and and the hard x-ray observations. The division has a physical basis. Soft x-ray flares correspond to thermal emission, while hard x-ray flares represent emission from accelerated particles.

#### Distributions from many active regions

The first frequency distributions of flare peak fluxes in soft X-rays were reported from OSO-3 observations in the energy range 7.7-12.5 keV in Hudson et al. (1969). They found a powerlaw slope of \( \alpha \approx 1.8 \) in the frequency distribution.

Drake (1971) used Explorer 33 and 35 data from years 1966-1968 in the wavelength range 2-12Å and found powerlaw distributions with exponents \( \alpha_P = 1.75 \) for the soft x-ray peak flux and \( \alpha_E = 1.44 \) for the fluence (peak \( \times \) duration).

Shimizu (1995) analyzed 5,000 transient brightenings from a single small active region using the YOKHOH SXT. He found \( \alpha_P = 1.64-1.89 \) and \( \alpha_E \approx 1.5-1.6 \), depending on the spatial area used in the sampling.

In a similar study Shimojo et al. (1999) analyzed microflares from a miniature active region and found \( \alpha_P = 1.7 \pm 0.4 \).

Feldman et al. (1997) studied 1,000 flare events in 1-8Å and found \( \alpha_P = 1.88 \pm 0.21 \).

Veronig et al. (2002) studied 50,000 flares from the GOES XRS and obtained \( \alpha_P = 2.11 \pm 0.13 \) for the peaks, \( \alpha_E = 2.03 \pm 0.09 \) for the fluence and \( \alpha_T = 2.93 \pm 0.12 \) for the durations. Yashiro et al. (2006) also report similar \( >2 \) values. These studies though have been accused of not performing any kind of background subtraction, which causes small events to have overestimated peak flux values.

#### Distributions from individual active regions

Sammis (1999) compared the distributions from two datasets:

1. all GOES soft x-ray flares recorded in the years 1989-1997, a total of 18,215 flares
2. a subset of the first dataset. Only active regions that produced at least one ≥ X1-class flare were selected and all the flares they produced were used in the dataset.

Table 1.1 below summarizes several studies and their corresponding slopes.

![Figure 1.1: Soft X-ray frequency distributions taken from Aschwanden (2011)](image)

<table>
<thead>
<tr>
<th>Powerlaw slope of peak flux $\alpha_p$</th>
<th>Powerlaw slope of total fluence $\alpha_E$</th>
<th>Powerlaw slope of durations $\alpha_t$</th>
<th>log range</th>
<th>Instrument</th>
<th>Reference</th>
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<td>1.8</td>
<td>1.44</td>
<td></td>
<td>1</td>
<td>OSO-3</td>
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<tr>
<td>1.75</td>
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<td></td>
<td>2</td>
<td>Explorer</td>
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<td>1.64–1.89</td>
<td>1.5–1.6</td>
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<td>Yohkoh</td>
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<td>1.86</td>
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<tr>
<td>1.88 ± 0.21</td>
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<td></td>
<td>3</td>
<td>GOES</td>
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<tr>
<td>1.7 ± 0.4</td>
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<td>Yohkoh</td>
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<tr>
<td>1.98</td>
<td>1.88</td>
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<td>GOES</td>
<td>7,8</td>
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<tr>
<td>2.11 ± 0.13*</td>
<td>2.03 ± 0.09*</td>
<td>2.93 ± 0.12*</td>
<td>3</td>
<td>GOES</td>
<td>8</td>
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<tr>
<td>2.16 ± 0.03*</td>
<td>2.01 ± 0.03*</td>
<td>2.87 ± 0.09*</td>
<td>3</td>
<td>GOES</td>
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* No background subtracted.

The comparison of the two distributions using a $\chi^2$ statistical test shows they are inconsistent. Sammis (1999) concludes that distributions of individual active regions are inconsistent with global distributions. Also different active regions do not share a common (not in the sense of equal but in the sense of statistically common) powerlaw. Sammis (1999) was the first to suggest that different regions can have different statistical signatures, thus global flare studies might be misleading.

Wheatland (2010a) analyzed AR11029, a small yet highly flare productive active region that appeared on the Sun at a time of low solar activity. The region produced around 70 small flares (all below C2.2). He constructs the cumulative distribution for this region and finds slopes < 2 and departure from powerlaw behavior. Using statistical tests he proves that the cum.dist. strongly favors a powerlaw plus rollover over a simple powerlaw. There is absence of large events and the cum.dist. ‘turns’ at a rollover halfway through the cum.dist. Wheatland (2010a) was the first to suggest that the distribution of a single region might not be a clear powerlaw from the smallest to the biggest flare of the region.

1.7 Self-Organized Criticality and Solar Flares

The discovery of powerlaws in the peak flux, fluence and also duration distributions of so many observational studies, of soft and hard x-ray flares, demands a theoretical explanation. The theory that is able to explain powerlaws is Self-Organized Criticality, pioneered by Per Bak, Chao Tang and Kurt Wiesenfeld in their seminal papers Bak et al. (1987) and Bak et al. (1988).
Chapter 1. Introduction

What BTW actually did, is discover a physics-independent, statistical way to describe the complex, nonlinear transport dynamics of certain classes of coupled systems. These systems are comprised of huge numbers of constituents that interact locally in a supportive yet simple manner. Despite the local simplicity of their interactions, as a whole the system displays complex behavior that cannot be understood by studying the individual components in isolation and then combining them to reproduce the whole (the classical superposition principle in physics). These systems are complex exactly because ‘the whole is much more than the parts’, meaning that working together, the components construct a system of great variability as opposed to their individual simplistic behavior.

BTW formulated their theory in terms of an algorithm of algebraic energy redistribution rules that describe the local interactions. The simulation of these rules in the form of a cellular automaton model on a computer, reveals the existence of self-organization within the system. As stated in Bak et al. (1988) : ‘Remarkably, the systems evolve naturally toward a critical state, with no intrinsic time or length scale.’ Therefore the system ‘chooses’, if left to evolve on its own, to operate in small scales in such a way, so that as a whole it will be brought to this critical state. This state has a number of extraordinary characteristics that make it stand out from other similar critical states such as phase transitions. These are:

- no fine tuning of system parameters is needed to reach the SOC state, as opposed to the fine tuning of temperature needed in phase transitions. This property is crucial for the appearance of SOC in a spontaneous manner, in a wide variety of natural systems.

- the critical state is an attractor of the dynamics meaning it will be reached no matter what the initial conditions are, how much the system parameters are varied or the presence of quenched randomness.

- the critical state is dynamically stable. The term ‘dynamically’ means that the system is not in equilibrium in the classical sense of the word, its behavior is widely variable and it changes abruptly and intermittently. The term ‘stable’ means that the system will always return to the SOC state shortly after a perturbation has disturbed it.

- in the critical state, the system self-organizes its structure into clusters of all sizes (which can also be thought of as formed bonds between the individual components, connectivity nodes, groups, communication channels etc.). These clusters come in sizes from only one site-component up to sizes comparable to the whole system. Most importantly, the individual sites are not connected all together. Therefore, a disturbance at one single site will only be communicated to a certain number of connected sites, not all the sites of the system. This spatial organization of the system is characterized then by self-similar fractal structures and makes fractality and indicator of SOC.

- at the critical state, energy is dissipated at all length and time scales (due to the presence of clusters of all sizes) and the events are called avalanches. The distribution of avalanche sizes and of avalanche durations are power-laws. This characteristic also forms a connecting chain between SOC dynamics and turbulence - where dissipation occurs in vortices of all sizes - and raises questions concerning their relationship.
1.7. Self-Organized Criticality and Solar Flares

• once in the SOC state, a system will be disturbed when the energy stored at a local site exceeds an instability threshold. The existence of such a threshold is an expression of the nonlinear dynamics of the system and is established by common physical conditions throughout the system.

• while at the SOC state, the timeseries of the (instantaneous) energy dissipation rate has a ‘1/f’ power spectrum, known as flicker noise. ‘1/f’ actually has a broader sense here. It does not mean that all SOC systems will display a -1 exponent in their power spectrum, it just means anything except white noise $S(f) \sim f^0$. Pink or flicker noise $S(f) \sim f^{-1}$, red noise $S(f) \sim f^{-2}$, and black noise $S(f) \sim f^{-3}$ are all considered manifestations of SOC dynamics if detected in a system. Such powerlaw power spectra reveal a basic truth of SOC systems: While avalanche timescales span over 1 or 2 decades of values with no preferred timescale, the local interactions between individual sites in all these scales are the same. This is why a slope without spikes appears in the log-log representation ($log S \sim \beta log f$) of the power spectrum. In retrospect, it might be hard to grasp how ‘1/f’ noise, implying long temporal correlations, could occur for a system without long-range spatial correlations, as is the case of SOC systems. It seems though that the existence and the equal ‘weight’ of all timescales makes it possible.

• the presence and maintenance of a SOC state requires the existence of a slow, continuous driver and the ability of the system for fast energy dissipation. Energy is injected into the system at a slow rate, stored locally (and randomly) at system sites, and dissipated out of the system during avalanches. The durations of avalanches have to be much shorter than the typical timescale of the driver.

• as a consequence of the above characteristic, a system in the SOC state conserves energy not at every instant of time but in averaged time periods. If we calculate the energy injected and the energy dissipated over a long period of time (during which many avalanches occur and many typical timescales of the driver pass), they will be equal. Essentially, an important part of the injected energy is stored at system sites, making the conservation of energy exact at each instant of time only if the stored energy is included in the equation.

• a system in the SOC state is characterized by a number of critical exponents which are connected with each other by scaling relations. Such critical exponents are the powerlaw slope of the avalanche size distribution, the powerlaw slope of the avalanche duration and the powerlaw slope of the power spectrum.

• a system in the SOC state obeys ‘finite-size scaling’ just as equilibrium statistical systems at the critical point. This means that low and high cutoffs are expected in the above mentioned powerlaw distributions, as a result of the physically smallest and largest scale of the system, respectively.

The SOC theory was adapted and applied to solar flares for the first time by Edward Lu and Russell Hamilton in their papers Lu and Hamilton (1991) and Lu, Hamilton, et al. (1993). Many variations of the original Lu&Hamilton solar flare avalanche model have been constructed since then, such as the model of Zirker et al. (1993), of Vlahos et al. (1995), of Georgoulis et al. (1996), of Georgoulis et al. (1998), of MacKinnon et al. (1997) and of Norman et al. (2001).
1.8 The Lu& Hamilton SOC Cellular Automaton for Solar Flares

As stated in the abstract of Lu and Hamilton (1991), ‘We propose that the solar coronal magnetic field is in a self-organized critical state, thus explaining the observed power-law dependence of solar flare occurrence rate on flare size, which extends over more than five orders of magnitude in peak flux.’ In the place of avalanches, LH saw solar flares. In the place of clusters of all sizes, they saw UCSs of all sizes, distributed at random places on an active region’s magnetic field network. The driver is photospheric convective motions that randomly shuffle the footpoints of the coronal magnetic field, storing energy in the form of stress at random places on the field (Low et al. (1990)). The threshold, which when exceeded triggers the start of an avalanche, is now represented by magnetic instability criteria. One example is Parker’s tangential discontinuity angle $\theta_c$ (Parker (1988)), which when exceeded leads to rapid dissipation of stored energy in the transverse magnetic field and local field restructuring. But any magnetic instability could be used in principle.

1.8.1 Why Study Solar Flares Using Statistics?

Most importantly, LH realized that the SOC theory provides a way of statistically describing the flaring mechanism and obtaining concrete predictions for flare parameter distributions and correlations. This is not what one can say for analytical or numerical approaches that try to solve nonlinear partial differential equations in active regions, which are inherently multiscale systems both in length and in time. Although significant progress has been made in recent years, especially in the field of MHD simulations, these advanced simulations have only recently been able to reproduce statistical distributions of flare parameters. SOC Cellular Automata for flares are still the most efficient way to gain understanding of the large-scale dynamics and statistical properties of an active region and of the process of energy release in flares.

It seems that understanding the microphysics of the instabilities, although crucial for understanding the physics of energy storage and release in complex magnetized plasmas, might not hold the key to understanding the powerlaw distributions of flare parameters and the systemic, synergistic properties of the magnetic field. Solar physicists describe flaring activity using a statistical theory, SOC, and computer simulations in the form of CA.

This statistical approach is the one adopted in this study. We try to gain better insight into the inner workings of the flaring mechanism by looking at the observational distributions of flare parameters from individual active regions and comparing those with the SOC CA simulated distributions (mainly the Lu&Hamilton SOC CA).

1.8.2 LH Model Description

The Flare Model of Lu&Hamilton is described in detail in their article Lu, Hamilton, et al. (1993) and summarized here.

The model uses a 3D discrete simulation grid, on each point $i$ of which a value
for the vector field $\mathbf{F}_i$ is specified. This vector field $\mathbf{F}_i$ could be the spatial average of the continuous magnetic field $\mathbf{B}$ of an active region. Alternatively, it could be the vector potential $\mathbf{A}$ of the magnetic field, in which case adding $\delta \mathbf{F}_i$ corresponds to adding a twist to the magnetic field at point $i$. Each point $i$ has 6 nearest neighbors. The local “slope” at point $i$ is:

$$d\mathbf{F}_i = \mathbf{F}_i - \sum_{j=1}^{6} \frac{1}{6} \mathbf{F}_{i+j}$$

(1.4)

A grid point $i$ is unstable when the magnitude of the slope at $i$ is greater than a specified critical value, $|d\mathbf{F}_i| > F_c$. When an instability occurs at $i$, the vector field is readjusted locally according to a specific set of rules that conserve both $\mathbf{F}$ and $d\mathbf{F}$. Furthermore:

- the boundary condition used is $\mathbf{F} = 0$ everywhere outside the boundary of the grid, meaning that LH is a closed boundary simulation.
- the system is driven by adding a random vector $\delta \mathbf{F}$ at a randomly selected grip point.
- the driver is weak $|\delta \mathbf{F}| \ll F_c$
- the driver is slow, meaning that flares have a much smaller duration in comparison to the typical timescale of photospheric motions that drive the instabilities.
- an elementary instability releases an amount of energy which is taken out of the field energy.
- each avalanche is characterized by its size $E$ (total number of unstable points), duration $T$ (total number of time steps from first instability to total relaxation), and peak activity $P$ (maximum number of unstable points in a given time step).
- after millions of iterations, the grid reaches a statistically steady state where the distributions for $E, T$ and $P$ become stationary. The active region is then in the SOC state.

### 1.8.3 LH Frequency Distributions

The frequency distributions of avalanche parameters obtained for grids of varying sizes are powerlaws over a large range of avalanche sizes (Table 1.3). The distributions have the following characteristics:

- all distributions can be fitted to a powerlaw with an exponential rollover (representing the upper cutoff):

$$N(X) = X^{-\alpha_x} \exp(X/X_c)$$

(1.5)

where $X$ stands for $E, P$ or $T$ and $X_c$ stands for their respective cutoffs. The $D(E)$ and $D(P)$ distributions have a well-defined powerlaw form for more than three and two orders of magnitude respectively. The $D(T)$ though is more problematic and can be fitted by a powerlaw for no more that one order of magnitude, introducing uncertainty in the values of the exponent $\alpha_T$. 

• there is a lower and an upper cutoff in all distributions, due to the smallest and largest scale of the grid respectively. Rescaling the distributions of the varying sized grids in order to plot them together shows that they overlay. The avalanche parameter distributions are therefore insensitive to the size of the grid for large enough grids, meaning that the difference in the fitted slopes of Table 1.3 are not statistically significant.

• changing the driving rate (making the driver not so slow, or even making it fast with a timescale comparable to the instability timescale) simply changes the waiting time between avalanches, without affecting the avalanche properties or the distributions.

Variations in the expectation value of the random added vector $\delta F$ affect flaring. If the expectation value of $\delta F$ is zero, the system never reaches the SOC state with the associated powerlaw distributions of avalanches. It is still avalanching, but the events are primarily small. Distributions obtained from a simulation with a driver that has an expectation value of $\delta F = 0$ are shown in Figure 1.2.

**Figure 1.2:** Freq.Dist. for E,P and T for a mean driving rate set to 0

Charbonneau et al. (2001a) also mention that before reaching the SOC state, the system is producing avalanches with sizes that increase gradually as the mean field and lattice energy grow. Once the SOC state is reached, a sudden increase in the size of the largest avalanches is observed, which now begin to span the whole
lattice. The timeseries before the SOC state resemble the timeseries of the SOC state.

**FIGURE 1.3:** Powerlaw indices obtained by Lu& Hamilton in their simulations

<table>
<thead>
<tr>
<th>$L$</th>
<th>$\alpha_E$</th>
<th>$E_e$</th>
<th>$\alpha_F$</th>
<th>$P_e$</th>
<th>$\alpha_P$</th>
<th>$T_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.40</td>
<td>521</td>
<td>1.29</td>
<td>11</td>
<td>0.89</td>
<td>23</td>
</tr>
<tr>
<td>15</td>
<td>1.46</td>
<td>2421</td>
<td>1.63</td>
<td>33</td>
<td>1.41</td>
<td>66</td>
</tr>
<tr>
<td>20</td>
<td>1.47</td>
<td>7416</td>
<td>1.72</td>
<td>64</td>
<td>1.60</td>
<td>131</td>
</tr>
<tr>
<td>25</td>
<td>1.45</td>
<td>$1.41 \times 10^4$</td>
<td>1.82</td>
<td>123</td>
<td>1.68</td>
<td>250</td>
</tr>
<tr>
<td>30</td>
<td>1.48</td>
<td>$2.79 \times 10^4$</td>
<td>1.88</td>
<td>197</td>
<td>1.66</td>
<td>266</td>
</tr>
<tr>
<td>40</td>
<td>1.49</td>
<td>$1.13 \times 10^5$</td>
<td>1.81</td>
<td>257</td>
<td>1.80</td>
<td>689</td>
</tr>
<tr>
<td>50</td>
<td>1.51</td>
<td>$2.85 \times 10^5$</td>
<td>1.86</td>
<td>470</td>
<td>1.88</td>
<td>1453</td>
</tr>
</tbody>
</table>

$\beta_E = 3.9$, $\beta_F = 2.3$, $\beta_P = 2.5$

### 1.8.4 LH Correlations Plots

LH find no correlation between the size $E$ of an avalanche and the waiting time between this avalanche and the one preceding or following it. Despite the lack of correlation in waiting times, all other avalanche parameters are correlated by well-defined relationships:

\[
E \propto P^{1.82} \quad \text{(1.6)}
\]

\[
E \propto T^{1.77} \quad \text{(1.7)}
\]

\[
P \propto T^{0.90} \quad \text{(1.8)}
\]

### 1.8.5 LH Model Predictions

LH make specific predictions that can be tested directly with solar flare observations:

1. The powerlaw indices of the flare frequency distributions will be the same for different active regions. Any differences in the indices will be statistically unimportant.

2. The observed flare duration distribution will be the most difficult to compare with the simulated one, due to the inability of correct flare duration measurements.

3. The power spectrum of the energy release rate will be a power law $S(f) \propto f^{-\beta}$, where $\beta \approx 2$

4. LH find $\alpha_E$ slopes less than 2, in contrast to Parker’s conjecture. This poses a problem for nanoflare heating. A possible mechanism that reconciles the two is the double powerlaw picture proposed by Georgoulis and Vlahos in Georgoulis et al. (1998), where SOC behavior and greater than 2 slopes are obtained with the use of anisotropic rules.

5. No appreciable correlation is expected between the energy of a flare and the time interval until the occurrence of another flare or between the energy of a flare and the time elapsed since the previous flare.
1.9 Solar Flare Waiting Time Distributions

1.9.1 Why are solar flare waiting time distributions important?

Solar flare waiting time distributions are not just another type of distribution that we construct just to argue for or against the presence of SOC in a system. WTDs are intrinsically related to the driver and its rate of energy input into the system. We study them to understand the driver and its characteristics. As stated in Aschwanden and McTiernan (2010): 'The statistics of waiting times bears crucial information on how a system works, either having independent elements that act randomly, or consisting of elements with long-range connections that enable coupling and synchronization.'

1.9.2 SOC WTDs

Events in SOC systems occur randomly in time and are thus independent of one another. Such events form an average constant stream, like the flow of electric current. They come at a constant average rate \( \lambda_0 \), corresponding to a constant average waiting time \( \langle w_t \rangle = \frac{1}{\lambda_0} \). Consequently, the WTD of flares is a Poisson distribution:

\[
P(w_t) = \langle w_t \rangle^{-1} \exp \left( -\frac{w_t}{\langle w_t \rangle} \right)
\]

This is an exponentially decaying distribution. Taking the logarithms of both sides of 1.9:

\[
\log P = \log \langle w_t \rangle^{-1} - \frac{\log e}{\langle w_t \rangle} w_t
\]

\[
\Rightarrow Y = b + aX
\]

where \( Y = \log P, X = w_t, a = -\frac{\log e}{\langle w_t \rangle} \) and \( b = \log \langle w_t \rangle^{-1} \).

Plotting a Poisson distribution on log-lin axis, a line should appear. SOC CA simulations for solar flares do show such Poisson distributions.

1.9.3 Are exponential WTDs an indisputable SOC feature?

In the first years following the formulation and simulations of SOC theory, exponential WTDs were considered a necessary SOC signature. In recent years though, it has been both analytically and computationally proven that the inherent time independence of the events in SOC systems, can result in other distributions. This understanding came about because the influence of time-varying drivers on SOC dynamics was investigated. From a computational viewpoint, Norman et al. (2001) used a non-stationary driver (random walk variations), and obtained WTDs with powerlaw tails. They propose that such WTDs can be understood in terms of a piecewise constant Poisson process.

From an analytical point of view, Wheatland (2000) and Wheatland et al. (2002a) formulate a time-dependent Poisson distribution with a time-varying flaring rate \( \lambda(t) \), equivalent to a time-varying driver \( \lambda(t) \rightarrow \langle w_t(t) \rangle^{-1} \rightarrow \text{driving rate}(t) \). They analytically show how a piecewise-constant Poisson process combined with exponentially distributed flaring rates produces a WTD that is a powerlaw for long waiting times. They propose that SOC is present in solar flares but the driver is non-stationary.
1.9.4 WTDs from Observations

Special issues with flare waiting time observations

There are four special issues that are known to cause problems with the construction and interpretation of WTDs from flare observations:

1. overlapping flares due to the lack of time scales separation in active regions. Although there is strong evidence that active regions are slowly-driven SOC systems (slowly-driven SOC CA produce powerlaws just like in flare observations) and therefore time separation should be granted, flare time-series reveal the existence of overlapping flares, in the form of flares that start during the cooling phase of previous flares, or even during the rise phase of large flares, or even in the form of double or triple peaked flares. This is especially true during periods of solar maximum but inspection of a typical flare timeseries would convince anyone that overlapping flares exist even during quiescent times. These lead either to an underestimation of the smallest durations and waiting times or to an overestimation of the largest durations and waiting times. Flare sympathy has also been observed and such an effect also produces overlapping flares.

2. different pulse detection methods. Specifically, Buchlin et al. (2005) compare 3 methods for extracting flare events from a timeseries, a peak detection method, a threshold method, and a wavelet method, and they show that the WTDs produced by these 3 methods differ in their shape and indices.

3. different event definitions. Sánchez et al. (2002) investigate the effect of defining a waiting time as:
   - the time from the end of a flare to the start of the next one
   - the time between consequent flare peaks
   - the time from start of a flare to the start of the next one

   and find that only the first definition produces exponential WTDs, given the same flare timeseries (the timeseries are produced from a 1-D SOC CA).

4. instrumental thresholds in flare flux/count rate measurements. The lack of flare observations of fluxes less than the instrumental threshold leads to the absence of small flares and thus small waiting times in the datasets. This is shown to affect the corresponding distributions (Buchlin et al. (2005), Hamon et al. (2002)).

Such limitations have to be kept in mind when we try to interpret WTDs from observation, compare WTDs from different observational studies or compare observed to simulated WTDs.

WTDs for flares from individual active regions

WTDs from individual active regions have been studied in two papers, Wheatland (2001) and Moon et al. (2001):
**Wheatland (2001)** examine the observed flaring rates, the rate variations, and the WTDS for flares from individual active regions. Wheatland uses 18 years of GOES soft x-ray data alongside the USAF/MWL active region observations, in order to extract only flares above C1 class that were identified with an active region of origin. Although he presents evidence for obscuration (large flares obscure smaller), and selection effects (large flares are more probable to be identified with an active region), his observations still show a substantial number of active regions with time-varying flaring rates. He concludes, based on a number of statistical tests and arguments, that the observational limitations cannot be artificially producing the rate variations. Wheatland uses a Bayesian procedure to determine the flaring rate in each region and from these rate or rates he constrains an appropriate constant or piecewise constant Poisson model. Examples of regions with and without rate variations are presented, and this difference of one or multiple rates is also seen in the form of the WTDS.

**Moon et al. (2001)** analyze soft x-ray flares stronger than C1 in a 2-year period of solar maximum. The WTD for the whole data is well represented by a piecewise-constant Poisson process. The flaring rate varies with a timescale of 2-3 days, based on a Kolmorogov-Smirnov test. 6 individual active regions are analyzed too, and their WTDS are fitted with a single rate Poisson process.

### 1.10 Solar Flare Power Spectra

Suppose we have a timeseries \(X(t)\) describing the time evolution of a system, plotted on a graph with \(t\) values on the x-axis and \(X(t)\) values on the y-axis. Suppose also that we split the t-axis into equal time intervals of some value \(T\). These equal intervals of time \(T\) constitute periods \(T\) time apart and correspond to a frequency of \(f=1/T\). Imagine now that we mark these intervals on the t-axis \((0,T,2T,...)\) and observe the values of the timeseries \(X(t)\) at these \(0,T,2T,...\) points. If they are all close to zero, wouldn’t it be reasonable to say that points in the timeseries which are period \(T\) apart don’t contribute significantly to the timeseries? Imagine now the opposite situation, where most values at a period \(T\) apart are observed to have moderate to high values on the X-axis. Wouldn’t it be reasonable to say that points in the timeseries \(T\) period apart contribute significantly to the timeseries? This is exactly what a power spectrum of a timeseries does. It measures the amount of significance of contribution of each period \(T\) and equivalently of each frequency \(f\).

In power spectra language: The power spectrum describes the distribution of power of a timeseries into frequency components. According to Fourier analysis, any physical signal can be decomposed into a number of discrete or continuous frequencies and the power of each frequency is a measure of the presence of the frequency in the original timeseries. For SOC systems, it has been established that their power spectra display powerlaw behavior: \(S(f) \sim f^{-\beta}\).

Many observational studies have determined the power spectrum of timeseries from various emissions produced by various features in the solar corona. From all these, the studies of UeNo et al. (1997) and Ireland et al. (2014) are relevant to the type of spectra created in this work. The findings of these two articles are summarized below.
1.11. Temporal Correlation Functions as a tool for the detection of SOC

UeNo et al. (1997) analyze soft x-ray GOES data obtained in 1991-1994, about 32 months in total, using 10min averages. They find that a typical power spectrum is comprised of:

1. a first flat part with $\beta_1 \sim 0$
2. a second moderate decline part with $\beta_2 \sim f^{-1}$
3. a third steep decline part with $\beta_3 \sim f^{-1.5}$ to $f^{-2}$

Break frequencies separating the three parts are at around $f \simeq 10^{-5}$ Hz and $f \simeq 10^{-4}$ Hz.

Ireland et al. (2014) analyze 6 hours of SDO AIA image data in the wavebands of 171 and 193. They create spectra corresponding to a quiet-Sun region, one that lies on top of a sunspot, a footpoint region, and a coronal moss region. The powerlaw indices for all regions and both wavebands average around a slope of $-2$, in the frequency range $10^{-5} - 10^{-2}$ Hz.

1.11 Temporal Correlation Functions as a tool for the detection of SOC

As stated in McAteer, Aschwanden, et al. (2016): "Every SOC researcher ultimately reverts back to the same set of unanswered questions - How can I tell whether my system is truly SOC, or if it is just displaying SOC-like behavior? The route to answering such questions begins with studies of autocorrelation functions."

Temporal autocorrelation measures the similarity of a signal with itself at different points in time. For example, given a timeseries of a periodic wave obscured by noise, the autocorrelation function will reveal the presence of the hidden periodicity ([https://en.wikipedia.org/wiki/Autocorrelation](https://en.wikipedia.org/wiki/Autocorrelation)). Correlation does not necessarily imply causation although it suggests it ([http://www.statsref.com/HTML/index.html?temporal_autocorrelation.html](http://www.statsref.com/HTML/index.html?temporal_autocorrelation.html)). The independence of observations in time cannot be inferred with certainty if the autocorrelation function shows lack of correlations, it can only be considered as a possible cause for the lack of correlations. Given a timeseries of equidistant observations $X_t$, a working definition for the temporal autocorrelation function is:

$$ACF(L) = \frac{\sum_{t=1}^{N-L}(X_t - \bar{X})(X_{t+L} - \bar{X})}{\sum_{t=1}^{N}(X_t - \bar{X})^2}$$  \hspace{1cm} (1.11)

where $L$ is the time distance between observations and takes values from 0 up to total time of observations. ACF(L) is the autocorrelation coefficient at time lag L, measuring the correlation of observations distant L time apart, and taking values in the range [-1,1]. ACF values are plotted against timelags L. To understand such a plot, the following basic notions are used:

- If the timeseries $X_t$ is random, the ACF curve will drop fast from $ACF(L = 0) \approx 1$ to zero and will be fluctuating in a narrow band around zero for all other timelags.
• For SOC systems, the appearance of the ACF curve resembles an exponential decay of the form \( ACF \propto \exp(-L/\tau) \), where \( \tau \) determines the correlation time and is equal to the value of \( L^* \) that makes \( ACF = 1/e \). This correlation time defines a time interval around every observation point, a kind of influence sphere in time, within which the observations have some degree of interdependency. We mostly pay attention to the behavior of the ACF curve before it reaches the value of 1/e.

• If the timeseries \( X_t \) has short term memory, it will start from \( ACF(L = 0) \approx 1 \) and quickly approach zero in an exponentially decaying fashion.

• If periodicities exist in the timeseries \( X_t \), these will appear as spikes.

• If the timeseries \( X_t \) has long term memory, the ACF curve will remain above 1/e for large timelags.

Crosscorrelation functions are used to measure the interdependencies between two different timeseries. Given a perturbation timeseries \( X_t \) and a response timeseries \( Y_t \), a working definition for crosscorrelation is:

\[
CCF(L) = \frac{\sum_{t=1}^{N-L} (X_t - \bar{X})(Y_{t+L} - \bar{Y})}{\sqrt{\left[ \sum_{t=1}^{N} (X_t - \bar{X})^2 \right] \left[ \sum_{t=1}^{N} (Y_t - \bar{Y})^2 \right]}} 
\]

(1.12)

where \( L \) is the time distance between observations in the \( X_t \) and \( Y_t \) timeseries and takes values from 0 up to the total time of observations. \( CCF(L) \) is the crosscorrelation coefficient at time lag \( L \), measuring the degree of dependency of the \( Y_t \) points to the \( X_{t-L} \) points for various timelags \( L \), and taking values in the range [-1,1]. CCF plots are analyzed in the same way as ACF plots. The crosscorrelation time is defined, describing a sphere of influence in time around a point \( X_i \) of the \( X_t \) timeseries. The points influenced by this ‘sphere’ are now all points inside a time interval \( L \) around the point \( Y_i \) of the \( Y_t \) timeseries.

1.12 The Scope of this study

1.12.1 Questions posed

This study aims to further our understanding on the following issues:

• Parker’s nanoflare heating requires frequency distributions with slopes >2. Such slopes are not found in distributions of flares from many active regions. Could it be though that >2 slopes exist in distributions of flares from individual active regions?

• Global distributions agree well with the LH simulations, implying the whole corona is in a SOC state. Do single region distributions agree with LH, implying they are also in a SOC state?

• The LH model makes predictions about the slopes of frequency distributions, flare parameter correlations, waiting times and fourier spectra powerlaws. Do these hold true for individual active regions?
• What is the form of WTDs for individual active regions? What could be inferred about the driver from the WTDs?

• What is the effect of studying flares from many regions together? Is there any effect at all?

• What can we tell about the solar coronal background? Could it be the product of unresolved nanoflares?

### 1.12.2 Methodology Outline

To address the above questions, active regions that acted mostly alone on the Sun’s visible side are identified. Lists containing the flares of these regions are constructed. Timeseries of the x-radiation produced by these regions are analyzed. With the help of the flare lists, the flare radiation and non-flaring background are separated. The thermal energy timeseries for the flares are created. The flare events, both in terms of x-rays and thermal energy, are statistically analyzed with cumulative distributions and scatter plots. The flare and background x-ray timeseries are analyzed with fourier plots and correlation functions.
Chapter 2

Data Collection

2.1 Compilation of the Flare Lists

The requirements for the flare lists of this study are:

1. they should consist of flares originating from only one active region
2. the active region should have produced more than 60 flares for the dataset to be statistically important
3. few other active regions can be present on the sun together with the selected active region but they should be significantly less flare productive

Such special lists do not generally exist. General flare lists report all flares observed and assign a start, stop, peak time and, if possible, a position and an active region number for each flare. Yet this last feature provides a way to extract from a general list the flares that came from a specific region. I exploit this feature and create single region flare lists from the general lists.

2.1.1 Choosing a general flare list - The LMSAL Archive

I investigated the flare lists mentioned below:

- the NGDC solar flare records (1955-2009) in the form of online monthly publications (https://www.ngdc.noaa.gov/stp/space-weather/online-publications/stp_sgd/)
- the YOHKOH flare catalogue (http://ylstone.physics.montana.edu/ylegacy/catalog_top.html)
- the RHESSI flarelist (http://hesperia.gsfc.nasa.gov/hessidata/dbase/hessi_flare_list.txt)
- the HINODE XRT flare catalogue (http://xrt.cfa.harvard.edu/flare_catalog/)
- the GOES X-ray Events Lists (http://hesperia.gsfc.nasa.gov/rhessidatcenter/complementary_data/goes.html)
- the LMSAL Latest Events Archive (http://www.lmsal.com/solarsoft/latest_events/)

I decided to use the LMSAL Archive for the following reasons:
1. the Flare Locator Image that accompanies the records provides the most possibly accurate determination of the position of origin of a flare. Other flare lists do not allow for visual inspection of events. The instruments used for the images are EUV and X-ray Imagers of the Sun such as the GOES SXI, SDO’s AIA and SOHO’s EIT.

2. this Archive reports the start, stop and peak time as determined by the GOES X-ray Flare Detection Algorithm. This is very convenient, because the statistical analysis in this study is done using the GOES XRS timeseries.

A typical record of an event in the archive is presented in Figure 2.1.

As described in http://www.swpc.noaa.gov/products/goes-x-ray-flux, the GOES algorithm identifies a start, stop and peak time based on the following criteria:

• the start time of an x-ray event is defined as the first minute, in a sequence of 4 minutes, of steep monotonic increase in 0.1-0.8 nm flux.
• the x-ray event maximum is taken as the minute of the peak x-ray flux.
• the end time is the time when the flux level decays to a point halfway between the maximum flux and the pre-flare background level.

Sometimes the algorithm will not trigger on a flare with a gradual rise-time (common for limb events), and a forecaster of NOAA will have to enter the particulars manually.

2.1.2 Searching for individual active regions

I decided to search for individual active regions from the decaying phase of the past solar cycle and onwards. To find the time, I consulted the SILSO sunspot number timeseries (http://sidc.oma.be/silso/), shown in Figure 2.2, and the Wolf number $S_n \sim (10g + s)$. Based on these, I searched the LMSAL Archive starting from January 2006.

In parallel I used SolarMonitor (https://www.solarmonitor.org/), where daily records of solar images from GOES SXI, HINODE XRT, SOHO EIT and SDO AIA can be found. Looking at these images, the search in the Archive was accelerated:

• sometimes regions appear concurrently on the Sun (as seen on magnetograms) but only one of them is X-ray productive (as seen from x-ray/euv imagers). The images helped me single out times when this happened.
• when many x-ray productive regions exist on the Sun, the images helped me determine when these rotate behind the west limb. The search in the list can continue after this time.

For periods of interest, the Archive was searched one daily record after the other in reverse chronological order. An individual active region candidate is found when in 3-4 consequent daily records most flares are assigned to a specific region. If the region continues its activity for the following 6-7 days at least, I proceed to compile the flare list of the region. Three special issues of the Archive must be considered:
2.1. Compilation of the Flare Lists

Figure 2.1: A typical event record of the LMSAL Archive

<table>
<thead>
<tr>
<th>Event#</th>
<th>EName</th>
<th>Start</th>
<th>Stop</th>
<th>Peak</th>
<th>GOES Class</th>
<th>Derived Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>gev_20160608_0404</td>
<td>2016/06/08 04:00:00</td>
<td>04:16:00</td>
<td>B2.3</td>
<td>N12W20</td>
<td>(2552)</td>
</tr>
</tbody>
</table>

1. Each daily record contains all flares that have occurred some days before the record’s compilation date. As a result in two consequent daily records some flares are the same. This is because past flares that have already been reported are reviewed again for position and active region assignment checks as new images are made available from the imaging instruments. When compiling my flare lists, I make sure I extract the latest reviewed version for each flare. I do this by extracting all flares from the beginning of a daily record up until the flare that appears first on the following day’s record. All flares that happened before this first flare will not be reviewed again and are to be trusted for position accuracy.

2. Some flares are assigned a position but not an active region number. This seems to be due to two reasons:
   - the flares come from an area where no named active region exists
   - the flares come from a named active region but for some reason they are not assigned to their region
I arrived to the above conclusions by checking the Flare Locator Image (see 2.1) for all flares in my lists that are not assigned a region. If the Flare Locator Image shows a non-assigned flare to originate from my selected target region, then the flare is included in my flare list.

3. Rarely, duplicate flares exist in the records. I define duplicate flares as two consequent flares with the second having a start time before the end of the first one. I choose to initially include the duplicate flares in the flare lists I compile. In the analysis of the lists, plotting of the timeseries allows for visual inspection of such duplicate events. This in turn allows for a more accurate manual determination of the proper start and end times or the rejection of wrong duplicates.

Finally, ten individual active regions were identified in the years 2006-2016. For each of these I compiled two lists:

- one containing only the selected region’s flares (flare list)
- one containing all flares from all regions (all events list)

Both lists start with the selected region’s first observed flare and end with the selected region’s last flare. A sample part of a list is shown in Figure 2.3. For each flare the following are recorded:

- date of occurrence
- start time in the form hours:minutes
- end time in the form hours:minutes
- peak time in the form hours:minutes
- size (e.x. C2.5)
- location (e.x. S05W63)
2.2 Retrieval of the GOES timeseries

The radiation signature of the events in the lists can be seen in various wavelengths. In this study we are interested in x-ray timeseries. We use those measured by the XRS instrument on board the GOES satellites because:

- GOES X-ray flux measurements have been made since 1986 in an almost uninterrupted way. This ensures that whatever the time range of an active regions activity, the GOES timeseries for it most likely exist.
- GOES timeseries can be easily retrieved through the SolarSoftWare (SSW) system.

### Figure 2.3: Sample part taken from the list of AR10960

- active region of origin

In Table 2.1 some information about the selected active regions are presented. I introduce q-value, a quality index showing the degree of flare productivity of the selected region with respect to the other concurrent regions.

\[
q = \frac{\#AR \text{ flares}}{\#all \text{ flares}} \quad (2.1)
\]

### Table 2.1: Information on the Selected Active Regions

<table>
<thead>
<tr>
<th>Active Region</th>
<th>Activity Period</th>
<th># AR flares</th>
<th># X</th>
<th># M</th>
<th># C</th>
<th># B</th>
<th>q-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>10848</td>
<td>14 - 29/1/2006</td>
<td>83/99</td>
<td>-</td>
<td>-</td>
<td>7</td>
<td>75</td>
<td>0.84</td>
</tr>
<tr>
<td>10930</td>
<td>3 - 18/12/2006</td>
<td>167/183</td>
<td>4</td>
<td>5</td>
<td>46</td>
<td>112</td>
<td>0.91</td>
</tr>
<tr>
<td>10960</td>
<td>31/5 - 1/6/2007</td>
<td>102/125</td>
<td>-</td>
<td>10</td>
<td>16</td>
<td>76</td>
<td>0.82</td>
</tr>
<tr>
<td>10963</td>
<td>7 - 18/7/2007</td>
<td>79/94</td>
<td>-</td>
<td>-</td>
<td>18</td>
<td>61</td>
<td>0.84</td>
</tr>
<tr>
<td>10978</td>
<td>3 - 19/12/2007</td>
<td>79/119</td>
<td>-</td>
<td>-</td>
<td>9</td>
<td>66</td>
<td>0.66</td>
</tr>
<tr>
<td>11029</td>
<td>24/10 - 1/11/2009</td>
<td>64/71</td>
<td>-</td>
<td>-</td>
<td>9</td>
<td>53</td>
<td>0.90</td>
</tr>
<tr>
<td>11158</td>
<td>11 - 21/2/2011</td>
<td>82/124</td>
<td>1</td>
<td>7</td>
<td>65</td>
<td>9</td>
<td>0.66</td>
</tr>
<tr>
<td>11271</td>
<td>13 - 30/8/2011</td>
<td>76/130</td>
<td>-</td>
<td>-</td>
<td>11</td>
<td>65</td>
<td>0.58</td>
</tr>
<tr>
<td>12192</td>
<td>14 - 31/10/2014</td>
<td>134/144</td>
<td>6</td>
<td>34</td>
<td>94</td>
<td>-</td>
<td>0.93</td>
</tr>
<tr>
<td>12529</td>
<td>4 - 21/4/2016</td>
<td>78/88</td>
<td>-</td>
<td>1</td>
<td>30</td>
<td>47</td>
<td>0.89</td>
</tr>
</tbody>
</table>
2.2.1 The GOES satellites and the XRS instrument

The GOES multimission program is comprised of a series of satellites from GOES-1 launched on 1975-10-16 to GOES-15 launched on 2010-03-04. They operate in geosynchronous orbits at 35,790 km above the equator in the west hemisphere of the Earth. More than one GOES satellite is at all times in space. GOES satellites monitor the weather and the space environment.

On each GOES satellite there are two X-ray Sensors (XRS) that provide solar X-ray fluxes for the wavelength bands of 0.5 to 4 (short channel) and 1 to 8 (long channel). These XRS are whole-disk X-ray photometers mounted on the solar array yoke on the side of the spacecraft body in a position that provides a clear field of view to the Sun at all times. The measurements are obtained from two gas-filled ion chambers, one for each band. Sweeper magnets deflect incoming electrons away from the assemblies so that only x rays are measured. The instruments are not thought to degrade.

Nominal flux levels expected are on the order of $2 \times 10^{-8}$ to $2 \times 10^{-3}$ W/m$^2$ for the long channel and $5 \times 10^{-9}$ to $5 \times 10^{-4}$ W/m$^2$ for the short channel. The data sampling rate depends on the satellite generation. For GOES 13-15, the data accumulation time is 2.048 s, and both the A and B channels take data simultaneously (Data and calibration handbook, 2009, Section 9.3). For satellites prior to GOES-13, the data accumulation time is 3 s. To get true fluxes for GOES 8-15 XRS, users are instructed to remove the SWPC scaling factors from the data, by dividing the short band flux by 0.85 and dividing the long band flux by 0.7. Removal of the scaling factors is not implemented in this work, since for a statistical study the important thing is how flare fluxes relate with each other, not their absolute value.

2.2.2 The SolarSoftWare (SSW) system

SolarSoft is basically a library that seats on top of an IDL installation. The SolarSoft tree is comprised of installation mandatory branches (SITE, GEN, SOHO, YOHKOH) and installation optional branches (mission specific libraries, general solar physics packages). All these contain directories and subdirectories where IDL routines exist.

For the needs of this study, SolarSoft was installed on phaethon.astro.auth.gr, a server of the Astrophysics, Astronomy and Mechanics Section of the Aristotle University Physics Department.

2.2.3 The process of timeseries retrieval

For the retrieval of the GOES x-ray timeseries the SolarSoft routine plot_goes_mkg.pro was used (~/SSW/goes/sxig12/SSWSXIIDL/plot_goes_mkg.pro). This routine was written by Dr. M.K. Georgoulis and implements the following:

- finds the GOES x-ray timeseries between a selected start and end time. It does this using the goes IDL class object and its methods (set and getdata) to automatically search for and retrieve GOES data files across the network for the time interval requested (http://hesperia.gsfc.nasa.gov/rhessidatapcenter/complementary_data/ goes.html). The getdata method extracts the data and derived quantities and places them into a structure. The structure (ar_number) contains many fields of which the following are used here:
2.2. Retrieval of the GOES timeseries

- ar_number.tarray : contains the second of observation for each data point starting with zero for the first data point
- ar_number.yclean[*,0] : contains the long channel’s flux values of the timeseries
- ar_number.yclean[*,1] : contains the short channel’s flux values of the timeseries

This structure is saved in an IDL .sav file and restored later for the analysis.

- plots the timeseries and saves the plot in an .png image file.

To create a .sav file for the timeseries of each flare list, I set:
- the start time equal to the start of the day of the first flare in the list
- the end time equal to the start of the day following the day of the last flare in the list

and run plot_goes_mkg ten times for the ten flare lists. A typical timeseries plot is shown in Figure 2.4.

For regions AR11271, AR12192 and AR12529 the timeseries are downloaded and saved but not plotted. Inspection reveals the existence of zero values in these timeseries. Specifically:
- the timeseries for AR11271 has zero values for approximately 15min
- the timeseries for AR12192 has zero values for approximately 139min
- the timeseries for AR12529 has zero values for approximately 347min

Lack of data points must be due to some instrumental failure or some problem with the telemetry. If flares occurred during these times, they are not recorded in the timeseries and in the lists. For the time range of AR11271 and AR12192 only GOES 15 was operational, therefore no better data are available. For the time range of AR12529 both GOES 13 and GOES 15 were operational, and I used GOES 15 data because they lack less points.
Chapter 3

Analysis and Results

3.1 Description of the analysis algorithm

The analysis algorithm was written in IDL (Version 8.0 running on linux x86_64 m64). Given a timeseries&event list pair it implements the following:

1. restores the .sav file containing the timeseries structure (ar_number)
2. decides whether the timeseries is a 3-sec complete, a 3-sec incomplete, a 2-sec complete or a 2-sec incomplete. Incomplete means that data points are missing.
3. if it is a 3-sec complete or a 2-sec complete: the algorithm just bins the data points into 1-min flux arrays for both the long and short channel
4. if it is a 3-sec incomplete or a 2-sec incomplete: the algorithm first corrects it and then bins it in 1-min flux arrays.
5. reads into IDL the flare list.
6. checks for duplicate flares in the list which I manually correct.
7. uses the date, start, stop and peak time of each flare to find its flux profile in the timeseries (long and short channel separately)
8. using this profile, it calculates for each flare (long and short channel separately):
   - a more accurate start, end and peak time
   - the background flux profile
   - the background-subtracted flux profile
   - the background-subtracted peak flux value
9. creates new timeseries (long and short channel separately) with zero flux values between flares and the background-subtracted flux profiles in place of the non background subtracted original flare profiles
10. calculates a temperature and an emission measure timeseries using these new flare timeseries and the SolarSoft routine goes_chianti_tem (/SSW/gen/idl/synoptic/goes_chianti_tem.pro) (long and short channel separately)
11. calculates the flare thermal energy timeseries using the temperature and emission measure timeseries of both channels
12. calculates the start and end of thermal energy release for each flare, the
duration and peak of thermal energy release and the total thermal energy
released during each flare

13. calculates the waiting time between flares

14. creates the following plots :
   - cumulative number distributions for the x-ray peaks
   - cumulative number distributions for the flare durations
   - cumulative number distributions for the waiting times
   - cumulative number distributions for the flare thermal energy
   - cumulative number distributions for the flare thermal energy peaks
   - waiting time vs. thermal energy correlation plots
   - thermal energy vs. thermal energy peaks correlation plots
   - thermal energy peaks vs. flare duration correlation plots
   - power spectrum plots
   - temporal correlation function plots for the flare and the background
timeseries
   - temporal response correlation function plots for the relation of the flare
to the background timeseries

15. finds linear fits for these plots. The fitting intervals are manually chosen
based on visual inspection.

   The IDL code can be found in Appendix A.

3.1.1 Why is the correction of data gaps needed ?

A 3-sec timeseries is expected to have as many data points as the seconds of the
days it covers, divided by 3. A 2-sec timeseries is expected to have as many
data points as the seconds of the days it covers, divided by 2. The sizes of the
structures in this study show the number of data points to be less than expected.
Also, ar_number.tarray reveal time jumps in consequent points.

Based on the above, I conclude that data gaps exist in the timeseries. It is
necessary to correct these gaps in order to find accurate 1-min averages. In the
flare lists, the event times are given in minutes. If the timeseries points are binned
wrongly, the start, stop and peak times are not accurately positioned inside the
timeseries.

The gaps are corrected by interpolation of a linear model between the original
flux value of the last point before the gap and the original flux value of first point
after the gap.

3.1.2 Checks for duplicate flares

There are three types of duplicate records and are all manually corrected :

   - records that clearly refer to the same event, but are registered two times,
     with slightly different start, stop or peak times. One of them is discarded.
• records with same start and stop but different peak times. One of them is discarded and the remaining is assigned a peak time that corresponds to the greatest of the two peaks.

• records of overlapping flares, where one starts before the other ends. Their start and stop times are corrected, so that at least a 0 waiting time can be calculated for them.

The duplicates found in the 10 flare lists compiled for this study are presented in Figure 3.1.

**Figure 3.1:** The duplicate flares in the lists from the LMSAL Archive

AR10930:

- gev_20061209_1352 2006/12/09 13:52:00 14:03:00 13:58:00 B1.6 S02E21 (930)
- gev_20061209_1353 2006/12/09 13:53:00 14:02:00 13:58:00 B1.4 S02E21 (930)

AR10963:

- gev_20070711_0915 2007/07/11 09:15:00 10:42:00 10:16:00 82.4 S07E89
- gev_20070711_1002 2007/07/11 10:02:00 10:32:00 10:29:00

AR11158:

- gev_20101221_1146 2010/12/21 11:46:00 11:51:00 11:51:00 B1.9 NL2E88 ()
- gev_20101221_1145 2010/12/21 11:46:00 13:11:00 12:23:00 B2.1 NL2E88 (1158)
- gev_20101221_1147 2010/12/21 11:47:00 11:57:00 11:54:00 B1.0 NL2E88 (1158)
- gev_20101221_0324 2010/12/21 03:24:00 05:08:00 04:48:00 C7.1 S02W00 (1158)
- gev_20101221_0429 2010/12/21 04:29:00 05:09:00 04:49:00 C8.3 S02W00 (1158)

AR12192:

- gev_20141025_0654 2014/10/25 06:54:00 08:50:00 08:23:00 C9.5 S16W25 (2192)
- gev_20141025_0736 2014/10/25 07:36:00 08:35:00 08:08:00 C9.2 S16W25 (2192)

AR12529:

- gev_20160409_1340 2016/04/09 13:40:00 15:10:00 13:42:00 C2.8 NL2E64 (2529)
- gev_20160409_1215 2016/04/09 12:15:00 14:44:00 13:42:00 C2.8 NL2E64 (2529)

3.1.3 Finding new start, end and peak times

If the start, stop and peak times are plotted on the timeseries, occasional discrepancies are revealed. There are cases where the visually expected times differ from those of the list. For example:

- In flares with a gradual rise phase, the start time from the list might be placed after the gradual rise phase.

- The stop time might be such that an important part of the cooling phase of the flare is neglected.

- On rare occasions, the peak time might not be set to the minute of maximum flux.

In this study I am interested in deriving the total thermal energy of a flare which is released during both the rise and the cooling phase. Thus these have to
be incorporated into the flare profile. Two different algorithms are written, one that finds the correct times for the long channel, and one that finds them for the short channel. The start, stop and peak times for the two channels are generally different, with the short channel’s times usually preceeding the long channel’s times. An example of the new times found for a long channel flare is shown in Figure 3.2. The flare list times are connected with red lines. The algorithm new times are connected with green lines.

Figure 3.2: Example flare with corrected times

3.1.4 The importance of background subtraction

The structure obtained with the plot_goes_mkg routine contains many fields. Two of them are the temperature and emission measure timeseries calculated by SolarSoft routines from the flux timeseries without background subtraction. When emission measure timeseries are plotted, a dip is observed in the beginning of all flares. As Wolfson et al. (1978) suggest, and as is widely accepted, the apparent emission dip is an artifact caused by the lack of background subtraction. In the initial rise phase the hot flare plasma is a small portion of the total cool active region plasma, with a corresponding smaller initial emission measure. Since the emission measure profile is flare plasma dominated, initially it will show a decrease and then a subsequent increase. Wolfson et al. (1978) show that a monotonically increasing emission measure is obtained when a preflare background is subtracted from the flare profile.

In Ryan et al. (2012) a preview of existing background subtraction methods is presented:

- No background subtraction: it is assumed that the total flux is flare dominated.
- SolarSoft GOES class object method: time intervals of preflare and afterflare flux levels are determined. A polynomial or a line is fitted between these two flux intervals and the fitted flux is subtracted from the total flux.
- Bornmann method: different methods are applied on a flare, and the one giving the most physically meaningful temperature and emission measure profiles is chosen.
3.1. **Description of the analysis algorithm**

- Wheatland (2010b) method: the duration of the rise phase of a flare is calculated. A single, steady background value is calculated, from the mean of as many points before the start as the length of duration. This value is subtracted from every flux point between the start and end time of the flare.

3.1.5 **Description of the background subtraction algorithm**

In this study, a new background subtraction algorithm was created, deriving from the methods mentioned above. It successfully produces emission profiles with no artificial dips.

My background subtraction algorithm implements the following for each flare:

- it creates a flare profile using the Wheatland (2010b) method.
- it creates a flare profile by interpolating a linear model between the flux at the start and the flux at the end.
- with both methods, sometimes for a few points in the beginning of a flare, the original flux is less than the background value. For these points the new flux is set to 0. This does not affect the analysis, because flare durations are not calculated from x-ray flare profiles but from the thermal energy profiles.
- it chooses the most appropriate method for each flare separately. The best method is the one creating the profile with higher flux values.

Flare timeseries were created for the long and short channel separately. These are zero valued between flares and in the place of flares have the background-subtracted profiles. 2 different flare timeseries were created for each channel, the flare timeseries and the all events timeseries.

An example of the background subtraction procedure is shown in Figures 3.3 and 3.4 (same flare in both channels).

- The black points are the original timeseries
- a red line connects the flare list start, peak, stop points
- a green line connects the new start, peak, stop points as identified by my algorithm
- if only a red line appears, my algorithm’s times and the list’s times agree.
- a green line that shows the linear interpolation background
- a blue line shows the Wheatland background
- an orange line shows the flare profile by subtraction of the linear interpolation
- the purple line show the flare profile by subtraction of the Wheatland background
Background timeseries were also created. These represent radiation from:

- **nanoflares.** These are really of an intermittent and bursty nature, but their speculated large numbers and high occurrence frequency result in ‘background’ looking radiation timeseries.

- **coronal waves.** Although these are most likely not energetic enough to heat the corona, they certainly contribute to the background coronal radiation.

- **non-solar radiation.** The GOES satellites are inside the outer radiation belt of the Earth’s magnetosphere (13-60 km), where populations of high energetic electrons and various ions exist. These could be producing low levels of non-solar x-radiation through a number of mechanisms.

Background timeseries were created for the long and short channel separately. These are equal to the original timeseries between flares and in the place of flares have the background values. Background timeseries were created from the all events lists only.
3.1. Description of the analysis algorithm

3.1.6 Calculation of temperature and emission measure timeseries

Thomas et al. (1985) have constructed analytic fits that can be used to determine temperatures and emission measures from GOES observations. Their method is based on the following:

The measured X-ray flux of channel $i$ (short : $i=4$, long : $i=8$), the emission measure $EM$, the channel’s transfer function $G_i(\lambda)$ and the radiation $f(T, \lambda)$ from an isothermal flare plasma at temperature $T$, are all related:

$$\text{flux}_i = EM \int_0^\infty G_i(\lambda) f(T, \lambda) \frac{d\lambda}{G_i} = EM \times b_i$$ (3.1)

The ratio $R$ of the two GOES channel fluxes is independent of $EM$ and only a function of $T$:

$$R(T) = \frac{\text{flux}_4}{\text{flux}_8} = \frac{b_4(T)}{b_8(T)}$$ (3.2)

For a temperature $T$:

The $f(T, \lambda)$ function is theoretically calculated (continuum+line emissions by McKenzie), and multiplied by the appropriate $G_i(\lambda)$ function. The $b_4(T), b_8(T)$ and $R(T)$ values can then be calculated as seen from Equations 3.1 and 3.2. This is done for different temperatures between $4 \times 10^6 K$ and $30 \times 10^6 K$. The $R(T)$ and $b_8(T)$ values are plotted against $T$ and the following analytic fits are determined:

$$T(R) = 3.15 + 77.2R - 164R^2 + 205R^3$$ (3.3)

where $T$ is in units of $10^6 K$.

$$10^{55}b_8(T) = -3.86 + 1.17T - 1.31 \times 10^{-2}T^2 + 1.78 \times 10^{-4}T^3$$ (3.4)

Given the measured flux values flux$_4$ and flux$_8$:

1. $R$ is calculated
2. Equation 3.3 is used to determine the temperature for this $R$
3. Equation 3.4 is used to determine the value of $b_8$ for this $T$
4. Due to Equation 3.1, dividing flux$_8$ by $b_8$ gives and value of EM (in units of $cm^{-3}$)

White, S. and Schwartz, R. have written the goes_chianti_tem routine, that computes temperatures and emission measures from GOES XRS measurements:

1. theoretical spectra by CHIANTI v7.1 with ionization equilibrium calculations due to Mazzotta et al. (1998) are used
2. the "FIP-effect" is considered, where coronal and photospheric abundances differ
3. coronal plasma density is $10^{10} cm^{-3}$
4. the temperature is computed from a spline fit from a lookup table for 101 temperatures, found in goes_get_chianti_temp.pro
5. the emission measure is derived from the temperature and $b_8$ using coefficients found in goes_get_chianti_em.pro
To get the correct transfer function, this routine requires knowledge of the GOES satellite that took the measurements. This is obtained through SolarSoft, using the goes object and the help method.

### 3.1.7 Calculation of thermal energy and relevant quantities

The thermal energy $E_T$ (in ergs) of an ideal gas of pressure $P$ (in dynes) and volume $V$ (in cm$^3$) is given by:

$$E_T = \frac{3}{2}PV \quad (3.5)$$

The pressure of a fully ionized hydrogen plasma (such as the coronal plasma), with pressure $P$ (in dynes), number density $n$ (in cm$^{-3}$) and temperature $T$ (in K), is given by:

$$P = 2knT \quad (3.6)$$

The emission measure EM (in cm$^{-3}$) of a gas is given by:

$$EM = \int n^2dV \quad (3.7)$$

For a homogeneous gas Equation 3.7 becomes:

$$EM = n^2V \quad (3.8)$$

Combining Equations 3.5, 3.6 and 3.8:

$$E_T = \frac{3kTEM}{n} \quad (3.9)$$

which is the thermal energy of a fully ionized hydrogen plasma that radiates away a quantity EM and is at a temperature $T$.

If we think of $T$ and EM of equation 3.9 as timeseries, as those produced from goes_chianti_tem, the calculated $E_T$ will also be a timeseries, that of thermal energy. Since the $T$ and EM timeseries are produced from flare timeseries, the $E_T$ timeseries will be the flare plasma thermal energy timeseries.

Thus, even though we are measuring flare radiation, which is only a part of the total energy released in a reconnection, we are able to ‘translate’ these measurements and obtain a good measure of the total energy released. This will be a lower limit of the true total energy released, because the energy given to particle acceleration is not considered. This though has a significant effect for large flares only, since in small flares acceleration is not important.

For each $T$ and EM timeseries, the $E_T$ timeseries was calculated from Equation 3.9 and using $n = 10^{10}$ cm$^{-3}$, as assumed in the goes_chianti_tem calculations.

One could raise the following objection: "Why go into such lengths to calculate the thermal energy of flares and not just use the peak soft x-ray flux or the fluence (peak flux $\times$ duration) as proxies for the energy released?"

The reason is we want to check observed distributions against the LH CA model. The predicted distributions are constructed from the energy release timeseries of the CA. Thus a proper comparison demands that flare energy timeseries are calculated from observations. This is why in the literature the standard way is using hard x-ray data and the thick target Bremsstrahlung model to derive
flare energies and compare with LH CA. I propose that using soft x-ray data and background subtraction to derive T and EM data is an equally valid way to get to flare energy timeseries and compare with LH CA.

Derivation of relevant quantities:

Using the $E_T$ timeseries the following relevant quantities are calculated for each flare:

- duration
- start of thermal energy release
- end of thermal energy release
- peak thermal energy release rate
- total thermal energy
- waiting time between flares

An example of a flare thermal energy profile is shown in Figure 3.5:

- the green points are the start and end times from the flare list
- the red points are the thermal energy start and end times, determined by my algorithm

**Figure 3.5: Thermal Energy Profile of a flare from AR10840**

3.2 Distributions

3.2.1 Construction

As far as num.dist. are concerned:
The number distribution of flares represents the number of flares observed per unit size $S$. Usually a powerlaw is fitted to a flare num.dist.:

$$N(S_k) = A S_k^{-\alpha} \quad (3.10)$$

A num.dist. can be constructed in two different but equivalent ways:

1. using logarithmic binning on the data values
2. using normal binning on the logarithms of data values

Logarithmic Binning:

- logarithmically spaced bins are created where the centres $S_k$ of the bins are non-equidistant and the binwidths become larger with increasing $S_k$:
  
  $$[10^{-7.0} - 10^{-6.9}], [10^{-6.9} - 10^{-6.8}], [10^{-6.8} - 10^{-6.7}], ..., [10^{-2.9} - 10^{-3.0}] \quad (3.11)$$

- the observations are placed inside the bins, giving $N(S_k)$
- the $N(S_k)$ values are plotted against the $S_k$ values on log-log axis

Taking the log of both sides of Equation 3.10:

$$\log N(S_k) = -\alpha \log S_k + \log A$$

$$\Rightarrow Y = -\alpha X + b \quad (3.12)$$

where $Y = \log N(S_k)$ and $X = \log S_k$. The fit on log-log axis is a line. The exponent $\alpha$ is calculated with least-squares/regression fitting.

Normal Binning of Logarithmic Data:

- for each flare of size $S_i$, $\log S_i$ is calculated
- the $\log S_i$ values are normally binned, giving $N(\log S_k)$
- the $N(\log S_k)$ values are plotted against the $\log S_k$ values on log-log axis

Substituting $S_k$ with $\log S_k$ on both sides of Equation 3.10 and taking the log of both sides:

$$N(\log S_k) = A(\log S_k)^{-\alpha}$$

$$\Rightarrow \log N(\log S_k) = \log A - \alpha \log(\log S_k)$$

$$\Rightarrow Y = -\alpha X + b \quad (3.13)$$

where $Y = \log N(\log S_k)$ and $X = \log(\log S_k)$. The fit on log-log axis is a line. The exponent $-\alpha$ is calculated with least-squares/regression fitting.

As far as cum.num.dist. are concerned:

For small datasets we work with cum.num.dist. and not with num.dist. Binning a small dataset results in a distribution with few points. Finding a fit from few points is problematic and uncertain. Cum.num.dist. prevail in this case, because each original data value is counted many times, artificially enhancing the
3.2. Distributions

size of the dataset. In cum.num.dist. there is no binning of the data and a fit is calculated from all original data values.

Usual cum.num.dist. in statistical analysis count how many observations have a size \( S \leq S_i \). Cum.num.dist. for flares are the opposite of that:

- the dataset of flares is sorted in order of increasing size \( S_1 < S_2 < S_3 < \ldots < S_{\text{max}} \)
- to calculate \( C(S_i) \), the number of flares with a size \( S > S_i \) must be counted. Using the sorted flare sequence, we count for each flare the number of flares after it. Thus, \( C(S_1) = \text{(number of flares in the dataset)} - 1 \) and \( C(S_{\text{max}}) = 0 \).
- the \( C(S_i) \) values are plotted against the \( S_i \) values on log-log axis.
- notice that \( C(S_{\text{max}}) = 0 \) cannot be plotted on a log y-axis and is consequently not used in the fit. This in no way harms the statistics, as \( S_{\text{max}} \) is counted ‘inside’ all points preceding it.

Mathematically, the construction of a cum.num.dist. which counts \( S > S_i \) is equivalent to the integration of the num.dist. from \( S \) to \( \infty \):

\[
N(S) = AS^{-\alpha} \\
C(S) = \int_{S}^{\infty} N(S)\,dS \\
\Rightarrow C(S) = \frac{A}{\alpha - 1} S^{-\alpha + 1}
\]

where \( \frac{A}{\alpha - 1} > 0, \alpha > 1 \) and the exponent \(-\alpha + 1 < 0\).

For the usual cum.dist. in statistics an integration from \(-\infty\) to \( S \) is used, and the resulting powerlaw has a negative proportionality constant. This explains why we use the opposite of the usual cum.dist.

Taking the log of both sides of Equation 3.16:

\[
\log C = \log \left( \frac{A}{\alpha - 1} \right) + ( -\alpha + 1 )\log S \\
\Rightarrow Y = ( -\alpha + 1 )X + b
\]

where \( Y = \log C \) and \( X = \log S \). The fit on log-log axis is a line. The exponent \( k = -\alpha + 1 \) is calculated with least-squares/regression fitting. The num.dist. exponent can be calculated as \(-\alpha = k - 1\).

In this study I am working with small datasets, with sizes from 64 to 183. Therefore, I use cum.num.dist. and construct the following distributions from observations:

- cum.num.dist. of x-ray peaks of the long GOES channel.
- cum.num.dist. of x-ray peaks of the short GOES channel.
- cum.num.dist. of total thermal energies of flares.
• cum.num.dist. of peak thermal energy release rate of flares.

• cum.num.dist. of durations of flares.

To find proper fits for the distributions, the fitting intervals are chosen manually based on the appearance of each distribution. For the chosen intervals linear fits are calculated using IDL least-square routines.

The above distributions are constructed 2 times for each active region, one with the dataset of events from the flare list and one with the dataset of the all events list.

3.2.2 Results

Distributions from the flare list datasets:

x-ray peaks

The distributions for the x-ray peaks of AR10848, AR10960, AR10963, AR10978, AR11029, AR11158, AR11271 and AR12529 show 2 distinct regions with different slopes. The second is always steeper than the first and greater than 2. The last points in the distributions of AR10848, AR10978 and AR11158 are not fitted by the second slope (outliers or third, even steeper slope). On the other hand, the distributions of AR10930 and AR12192 are well fitted by a single slope, which is always less than 2.

In Figures 3.6 and 3.7 representative plots are shown. Black points are the original long channel non-background-subtracted data. Blue points are the long channel background-subtracted data. Green points are the short channel background-subtracted data. The distributions for the rest of the active regions can be found in Appendix B.

**Figure 3.6: AR12529 : example x-ray peaks distribution with 2 distinct slopes**
The distributions for flare thermal energies can be split in 2 categories:

1. Category 1: The single slope active regions
   AR10930, AR10978, AR11158, AR12192 and AR12529 belong in this category. Their slopes are always less than 2. There are a few outliers in the end of the distributions.

2. Category 2: The double slope active regions
   AR10848, AR10960, AR10963, AR11029 and AR11271 belong in this category. The first slope is always less than 2 and the second always greater than 2.

In Figures 3.8 and 3.9 representative plots for the 2 categories are shown. In Table 3.1 the values of the slopes and their respective ranges are shown. The distributions for the rest of the active regions can be found in Appendix C.
Chapter 3. Analysis and Results

Figure 3.9: AR10963: example thermal energy distribution with 2 distinct slopes

Table 3.1: Thermal Energy Slopes and Respective Ranges

<table>
<thead>
<tr>
<th>Active Region</th>
<th>First Slope</th>
<th>Range (ergs)</th>
<th>Second Slope</th>
<th>Range (ergs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10848</td>
<td>1.55</td>
<td>$10^{31} - 2 \times 10^{32}$</td>
<td>2.27</td>
<td>$2 \times 10^{32} - 2 \times 10^{33}$</td>
</tr>
<tr>
<td>AR10930</td>
<td>1.43</td>
<td>$2 \times 10^{31} - 3 \times 10^{35}$</td>
<td>2.53</td>
<td>$7 \times 10^{32} - 5 \times 10^{33}$</td>
</tr>
<tr>
<td>AR10960</td>
<td>1.44</td>
<td>$2 \times 10^{31} - 3 \times 10^{33}$</td>
<td>2.53</td>
<td>$7 \times 10^{32} - 5 \times 10^{33}$</td>
</tr>
<tr>
<td>AR10963</td>
<td>1.33</td>
<td>$2 \times 10^{31} - 7 \times 10^{32}$</td>
<td>2.38</td>
<td>$3 \times 10^{33} - 2 \times 10^{34}$</td>
</tr>
<tr>
<td>AR10978</td>
<td>1.64</td>
<td>$2 \times 10^{31} - 10^{33}$</td>
<td>2.46</td>
<td>$3 \times 10^{32} - 2 \times 10^{33}$</td>
</tr>
<tr>
<td>AR11029</td>
<td>1.49</td>
<td>$2 \times 10^{31} - 3 \times 10^{32}$</td>
<td>2.46</td>
<td>$3 \times 10^{32} - 2 \times 10^{33}$</td>
</tr>
<tr>
<td>AR11158</td>
<td>1.65</td>
<td>$2 \times 10^{32} - 2 \times 10^{34}$</td>
<td>2.24</td>
<td>$5 \times 10^{32} - 5 \times 10^{33}$</td>
</tr>
<tr>
<td>AR11271</td>
<td>1.64</td>
<td>$9 \times 10^{31} - 5 \times 10^{32}$</td>
<td>2.24</td>
<td>$5 \times 10^{32} - 5 \times 10^{33}$</td>
</tr>
<tr>
<td>AR12192</td>
<td>1.41</td>
<td>$2 \times 10^{32} - 10^{35}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR12529</td>
<td>1.74</td>
<td>$2 \times 10^{32} - 2 \times 10^{34}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The distributions for the thermal energy peaks of AR10848, AR10963, AR11029 and AR11271 show 2 distinct regions with different slopes. The second is always steeper than the first and greater than 2. The distributions of AR10930, AR10960, AR10978, AR11158 and AR12192 are well fitted by a single slope, which is always less than 2. The distribution of AR12529 shows 3 slopes.

In Figures 3.10, 3.11 and 3.12 representative plots are shown. The distributions for the rest of the active regions can be found in Appendix D.
3.2. Distributions

The distributions for the duration of AR10848, AR10930, AR10960, AR10978, AR11158, AR11271 and AR12529 show 2 distinct regions with different slopes. The second is always steeper than the first and greater than 2. The distributions of AR10963, AR11029 and AR12192 show 3 distinct regions with different slopes. Each is steeper than the one preceding it. In Figures 3.13 and 3.14 representative plots are shown. The distributions for the rest of the active regions can be found in Appendix E.
Distributions from the all events list datasets:

The all events distributions, for all regions studied, are statistically indistinguishable from their corresponding flare distributions. Their form and slope values do not differ significantly. Due to this fact, they are not presented here. Three out of the 10 regions studied had concurrent regions with non-negligible contributions though. AR10978 and AR11158 both produced 66% and AR11271 produced 58% of the total flares observed, while all other flares came from other regions. Even for these regions the flare distributions and the all events distributions are statistically similar, as can be seen from comparison of the thermal energy plots in Appendix B.
3.3 Waiting Time Distributions and Model Comparisons

As discussed in the Introduction, a WTD with a single average waiting time will be a line on a log-lin plot. Similarly, I propose that a WTD with two or more different average waiting times will appear as two or more lines on a log-lin plot.

From the waiting times calculated by the analysis algorithm, WTDs were created and plotted on log-lin axis. For all active regions, except AR1158 and AR12192, the WTDS show two distinct lines. In Figures 3.15 and 3.16 representative examples are shown. The rest of the WTDS can be found in Appendix F. The values of the average waiting times and their errors are calculated from the fitted slopes $\alpha \pm \sigma_\alpha$ of the plots:

$$< w_t > = \frac{\text{loge}}{\alpha}$$

$$\sigma < w_t > = \frac{\text{loge}}{\alpha^2} \sigma_\alpha$$

They are presented in Table 3.2.

<table>
<thead>
<tr>
<th>Active Region</th>
<th>$&lt; w_{t1} &gt;$</th>
<th>$&lt; w_{t2} &gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10848</td>
<td>108.82 min</td>
<td>312.02 min</td>
</tr>
<tr>
<td>AR10930</td>
<td>102.93 min</td>
<td>258.16 min</td>
</tr>
<tr>
<td>AR10960</td>
<td>106.80 min</td>
<td>227.29 min</td>
</tr>
<tr>
<td>AR10963</td>
<td>26.63 min</td>
<td>64.48 min</td>
</tr>
<tr>
<td>AR10978</td>
<td>126.28 min</td>
<td>865.78 min</td>
</tr>
<tr>
<td>AR11029</td>
<td>83.11 min</td>
<td>280.93 min</td>
</tr>
<tr>
<td>AR11158</td>
<td>133.38 min</td>
<td></td>
</tr>
<tr>
<td>AR11271</td>
<td>191.34 min</td>
<td>554.97</td>
</tr>
<tr>
<td>AR12192</td>
<td>125.60</td>
<td></td>
</tr>
<tr>
<td>AR12529</td>
<td>268.04</td>
<td>287.25</td>
</tr>
</tbody>
</table>
Fig. 3.15: AR10960: example WTD with two average waiting times

Fig. 3.16: AR12192: example WTD with one average waiting time

Piecewise Constant Poisson Distributions

In the calculations below I am borrowing from Aschwanden and McTiernan (2010), Wheatland (2000) and Wheatland et al. (2002a).

If flares are random events in time that come at a mean constant rate $\lambda$, their WTD will be a stationary Poisson process:

$$P(\Delta t) = \lambda e^{-\lambda \Delta t}$$  \hspace{1cm} (3.20)

If flares are random events in time with varying rates $\lambda_i$ that are kept constant during time intervals $[t_i, t_{i+1}]$, their WTD will be a nonstationary Poisson process:

$$P(\Delta t) = \sum_i \phi_i \lambda_i exp(-\lambda_i \Delta t)$$  \hspace{1cm} (3.21)
where

\[ \phi_i = \frac{\lambda_i t_i}{\sum_j \lambda_j t_j} \]  \hspace{1cm} (3.22)

is the fraction of events associated with a given rate \( \lambda_i \).

**FIGURE 3.17:** Occurrence rates \( \lambda(t) \) and their corresponding WTDs taken from Aschwanden and McTiernan (2010)

If the rates vary continuously, the function \( \lambda(t) \) can be defined, and in this case (3.21) becomes:

\[ P(\Delta t) = \frac{\int_0^T \lambda(t)^2 e^{-\lambda(t)\Delta t} dt}{\int_0^T \lambda(t) dt} \]  \hspace{1cm} (3.23)
for a total time of observation $T$. Defining $f(\lambda) = (1/T)dt(\lambda)/d\lambda$, (3.23) can be replaced by:

$$P(\Delta t) = \frac{1}{\lambda_o} \int_0^\infty f(\lambda) \lambda^2 e^{-\lambda \Delta t} d\lambda$$

(3.24)

where

$$\lambda_o = \int_0^\infty \lambda f(\lambda) d\lambda$$

(3.25)

is the mean rate of flaring. If $\lambda(t)$ is given, $f(\lambda)$ is calculated, and by integration of (3.25) $P(\Delta t)$ is found.

In Figure 3.17 five different time-dependent flare occurrence rates $\lambda(t)$ are shown together with their corresponding WTDs:

1. a constant rate $\lambda_o$ produces a single exponential WTD
2. a two-step function with rates $\lambda_1$ and $\lambda_2$ produces a superposition of two exponential WTDs
3. a triangular pulse function $\lambda(t) = \lambda_o(t/T)$ produces a non-stationary Poisson WTD with a powerlaw tail of slope -3
4. an exponential pulse function produces a non-stationary Poisson WTD with a powerlaw tail of slope -3. Here, $f(\lambda)$ is:

$$f(\lambda) = \left(\frac{1}{\lambda_o}\right) exp\left(-\frac{\lambda}{\lambda_o}\right)$$

(3.26)

Substitution of (3.26) into (3.24):

$$P(\Delta t) = \frac{2\lambda_o}{(1 + \lambda_o \Delta t)^3}$$

(3.27)

In the limit of large waiting times ($\Delta t \gg 1/\lambda_o$), (3.27) becomes $P(\Delta t) \simeq \Delta t^{-3}$. This is a powerlaw, although the process is intrinsically random.

5. a $\delta$-pulse function produces a non-stationary Poisson WTD with a powerlaw tail of slope -2. Here, $f(\lambda)$ is:

$$f(\lambda) = \left(\frac{1}{\lambda}\right) exp\left(-\frac{\lambda}{\lambda_o}\right)$$

(3.28)

Substitution of (3.28) into (3.24):

$$P(\Delta t) = \frac{\lambda_o}{(1 + \lambda_o \Delta t)^2}$$

(3.29)

In the limit of large waiting times ($\Delta t \gg 1/\lambda_o$), (3.29) becomes $P(\Delta t) \simeq \Delta t^{-2}$. This is a powerlaw, although the process is intrinsically random.

**Application to the WTDs of this study**

As explained in the Introduction, the flaring rate $\lambda(t)$ is connected to the driving rate of active regions. When we are looking at a $\lambda(t)$ function, such as those of Figure 3.17, we are also looking at the functional form of the driver. As seen from Figure 3.17, if we have the WTD of a flare dataset, we can calculate fits on it, and
from them guess at the functional form of the driver of the region. This ‘inverse’ approach is used in this study to gain insight into the driver of the active regions studied.

To use WTDs in the way explained above, WTDs on log-log axis were constructed from the waiting times calculated by the analysis algorithm. The analytic functions that will be given below were then fitted on them. Specifically, since the datasets are small, cum.dist. WTDs were created. To compare these with model functions and find fits, Equations 3.20, 3.27 and 3.29 must be converted to their cumulative form.

Cum.dist. of a single rate Poisson distribution (model 3.20):

Integrating \( P(w_t) = ca exp(-aw_t) \), where \( c \) is a manually chosen normalization constant, and \( a = <w_t>^{-1} \):

\[
C(w_t) = \int_{w_t}^{\infty} P(w_t)dw_t = c.exp\left(-\frac{w_t}{<w_t>}\right) \tag{3.30}
\]

Taking the log of both sides of equation 3.30:

\[
logC = logc - loge\frac{w_t}{<w_t>} \Rightarrow logw_t = log\left(loge\frac{<w_t>}{loge}\right) + log(logC)
\]

\[\Rightarrow X = \text{const} + \log Y\]

where \( X = logw_t \) and \( Y = logC \).
Equation 3.30 was fitted on the log-log WTDs with a single average waiting time.

Cum.dist. of a double rate Poisson distribution:

The values \( <w_{t1}> \) and \( <w_{t2}> \), from the log-lin plots, can be used to construct two Poisson distributions:

\[
C_1(w_t) = c_1 exp\left(-\frac{w_t}{<w_{t1}>}\right) \tag{3.31}
\]
\[
C_2(w_t) = c_2 exp\left(-\frac{w_t}{<w_{t2}>}\right) \tag{3.32}
\]

Equations 3.31 and 3.32 were fitted on the log-log WTDs with two average waiting times. The normalization constants \( c_1 \) and \( c_2 \) were manually determined so that the functions 3.31 and 3.32 ‘meet’ the data in the appropriate height (since on log-log axis \( \text{const} = log\left(loge\frac{<w_t>}{loge}\right) \) determines the height of the curve).

The fitting intervals for each of the functions 3.31, 3.32 were also manually determined.

The standard procedure to construct piecewise constant Poisson functions in the literature is to use the method of Bayesian blocks. Since in my study regions show
1 or 2 distinct rates, I considered this unnecessary trouble and use the above technique.

**Cum.dist. for Wheatland’s distribution (model 3.27):**

I call this type of model ‘Wheatland’s distribution’, because Wheatland was the first to propose it. For calculation of the cum.dist.:

\[
P(w_t) = \frac{2 < w_t >^{-1}}{(1 + \frac{w_t}{< w_t >})^3} \tag{3.33}
\]

\[
\Rightarrow C(w_t) = \int_{w_t}^{\infty} \frac{2 < w_t >^{-1}}{(1 + \frac{w_t}{< w_t >})^3} dw_t \tag{3.34}
\]

\[
\Rightarrow C(w_t) = \frac{1}{(1 + \frac{w_t}{< w_t >})^2} \tag{3.35}
\]

where the substitution of \( a = 2 < w_t >^2 \) and \( b = < w_t > \) is convenient in the calculations.

Equation 3.35 was fitted on all log-log WTDs. The only parameter to be determined is the value of \( < w_t > \), which was set manually, based on which \( < w_t > \) makes equation 3.35 follow the WTD.

**Cum.dist. for Aschwanden’s distribution (model 3.27):**

I call this type of model ‘Aschwanden’s distribution’, because it was proposed by Aschwanden and McTiernan (2010). For calculation of the cum.dist.:

\[
P(w_t) = \frac{< w_t >^{-1}}{(1 + \frac{w_t}{< w_t >})^2} \tag{3.36}
\]

\[
\Rightarrow C(w_t) = \int_{w_t}^{\infty} \frac{< w_t >^{-1}}{(1 + \frac{w_t}{< w_t >})^2} dw_t \tag{3.37}
\]

\[
\Rightarrow C(w_t) = \frac{1}{1 + \frac{w_t}{< w_t >}} \tag{3.38}
\]

where the substitution of \( a =< w_t >^{-1} \) is convenient in the calculations.

Equation 3.38 was fitted on all log-log WTDs. The only parameter to be determined is the value of \( < w_t > \), which was set manually, based on which \( < w_t > \) makes equation 3.38 follow the WTD.

**Results:**

For most (6/10) of the active regions analyzed, the Poisson distribution (single or
3.3. Waiting Time Distributions and Model Comparisons

Double) is the best fit for the WTD. For the tail part of some (4/10) of the regions, Wheatland’s fit is better than Poisson’s. AR10978 is the only region that does not conform with the Poisson model and is better represented by Aschwanden’s fit.

In Figures 3.18, 3.19 and 3.20 representative WTDs are shown. The rest can be found in Appendix F. Table 3.3 summarizes which model is the best fit for each WTD.

Comments on the plots:

- The Poisson distribution fits are shown in red
- The Wheatland distribution fit is shown in purple
- The Aschwanden distribution fit is shown in blue
- The values of the average waiting times used for all fits are shown on the plot, using the same color code as for the fits

**Figure 3.18:** AR12529: example WTD where Poisson is best fit

**Figure 3.19:** AR10848: example WTD where Wheatland’s fit is best for the tail part
Chapter 3. Analysis and Results

FIGURE 3.20: AR10978: example WTD where Aschwanden’s fit is the best

<table>
<thead>
<tr>
<th>Active Region</th>
<th>Initial Part</th>
<th>Tail Part</th>
<th>On the whole</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10848</td>
<td>Poisson</td>
<td>Wheatland’s</td>
<td>-</td>
</tr>
<tr>
<td>AR10930</td>
<td>Poisson</td>
<td>Poisson</td>
<td>Poisson</td>
</tr>
<tr>
<td>AR10960</td>
<td>all</td>
<td>Poisson</td>
<td>Poisson</td>
</tr>
<tr>
<td>AR10963</td>
<td>all</td>
<td>Poisson/Wheatland’s</td>
<td>Poisson/Wheatland’s</td>
</tr>
<tr>
<td>AR10978</td>
<td>all</td>
<td>Aschwanden’s</td>
<td>Aschwanden’s</td>
</tr>
<tr>
<td>AR11029</td>
<td>Poisson</td>
<td>Wheatland’s</td>
<td>-</td>
</tr>
<tr>
<td>AR11158</td>
<td>Poisson</td>
<td>Poisson</td>
<td>Poisson</td>
</tr>
<tr>
<td>AR11271</td>
<td>Poisson</td>
<td>Wheatland’s</td>
<td>-</td>
</tr>
<tr>
<td>AR12192</td>
<td>Poisson</td>
<td>Poisson</td>
<td>Poisson</td>
</tr>
<tr>
<td>AR12529</td>
<td>Poisson</td>
<td>Poisson</td>
<td>Poisson</td>
</tr>
</tbody>
</table>

3.4 Fourier Analysis

In order to check whether the active regions of this study are in a SOC state, I construct the power spectra of the thermal energy timeseries. These are the closest measure to an active region’s flare energy dissipation rate. I also construct:

- power spectra of long channel flare timeseries
- power spectra of short channel flare timeseries
- power spectra of long channel background timeseries

The spectra are constructed using IDL’s built-in Fast Fourier Transform (FFT) function. As explained in IDL’s online documentation pages (https://www.harrisgeospatial.com/docs/fft.html), the FFT function uses a multivariate complex Fourier Transform. It calculates the complex, discrete Fourier
Transform $F(u)$ of an N-element, one-dimensional function $f(x)$, defined mathematically as:

$$F(u) = \frac{1}{N} \sum_{x=0}^{N-1} f(x) \exp[-j2\pi ux/N]$$  \hfill (3.39)

In practice, the FFT function receives as input an N-element array, in my case the $E_T$ timeseries or the flare timeseries of the two channels, which are all of sizes equal to the total number of minutes of the timeseries. The size of the input array does not necessarily have to be an integer power of 2, which is convenient given the variable size of the $E_T$ timeseries.

The FFT function converts the input array $f(x)$ to complex, and computes its Fourier Transform, the complex array $F(u)$, of equal size with $f(x)$. Element 0 of $F(u)$ contains the zero frequency component. Element 1 contains the smallest nonzero positive frequency equal to $\nu(1) = \frac{1}{NT}$, where $T$ is the sampling interval, in this study $T = 60\text{sec}$, because the timeseries used contain 1-min averaged values. Element 2 corresponds to a frequency of $\nu(2) = \frac{2}{NT}$, etc...Negative frequencies are stored after the greatest positive frequency, namely the element corresponding to the Nyquist critical frequency $\nu(\text{max}) = \frac{1}{2T}$.

The quantity plotted is the square of the absolute of each value of the computed complex array $F(u)$ (absolute $= \sqrt{\text{Real}^2 + \text{Imaginary}^2}$). To plot these values against their corresponding frequencies, the following frequencies are computed for each element $i$:

$$\nu(0) = 0$$
$$\nu(i) = \frac{i}{60 \times \text{num}\_\text{minutes}}, \text{ for } i=1,\ldots,\text{num}\_\text{minutes}/2$$  \hfill (3.40)

where num_minutes is the size of the $E_T$ and the flare timeseries.

### 3.4.1 Thermal Energy Power Spectra

Fits of the form $S(f) \sim f^{-\beta}$ are calculated on the spectra. For all regions except AR10930 and AR11158, the slopes are less than 2. Some spectra are clear powerlaws, like the one shown in Figure 3.21, while others are fluctuating, like the one shown in Figure 3.22. The rest can be found in Appendix G. In table 3.4 the fitted slopes with their corresponding ranges are shown.

### 3.4.2 Long Channel Background Power Spectra

Fits of the form $S(f) \sim f^{-\beta}$ are calculated on the spectra. For all regions, the slopes are close to 2. Unlike the thermal energy spectra, the background spectra for all regions are very well represented by powerlaws, without fluctuations. A representative plot is shown in Figure 3.23. The rest can be found in Appendix G. In table 3.5 the fitted slopes with their corresponding ranges are shown.
Chapter 3. Analysis and Results

**Figure 3.21:** AR10960: example of a clear powerlaw power spectrum

![Spectral Power of Thermal Energy Release Rate Timeseries of AR10960](image)

**Figure 3.22:** AR12192: example of a fluctuating power spectrum

![Spectral Power of Thermal Energy Release Rate Timeseries of AR12192](image)

**Table 3.4:** Thermal Energy Release Rate - Power Spectra Slopes

<table>
<thead>
<tr>
<th>Active Region</th>
<th>Powerlaw Slope</th>
<th>Frequency Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10848</td>
<td>1.29</td>
<td>$5.8 \times 10^{-5} - 3.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR10930</td>
<td>2.54</td>
<td>$1.4 \times 10^{-4} - 6.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR10960</td>
<td>1.47</td>
<td>$5.4 \times 10^{-5} - 8.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR10963</td>
<td>1.54</td>
<td>$9.6 \times 10^{-5} - 8.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR10978</td>
<td>1.31</td>
<td>$7.9 \times 10^{-5} - 5.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR11029</td>
<td>1.93</td>
<td>$8.5 \times 10^{-5} - 8.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR11158</td>
<td>2.97</td>
<td>$3.0 \times 10^{-4} - 8.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR11271</td>
<td>1.32</td>
<td>$5.1 \times 10^{-5} - 1.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR12192</td>
<td>1.86</td>
<td>$3.0 \times 10^{-5} - 3.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>AR12529</td>
<td>1.13</td>
<td>$2.9 \times 10^{-5} - 1.5 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
3.4. Fourier Analysis

**Figure 3.23:** AR10930: example of a background power spectrum

![Image of a background power spectrum for AR10930](image1)

- **Spectral Power of the 1-8A X-ray Background Timeseries of AR10930**

**Table 3.5:** Long Channel Background - Power Spectra Slopes

<table>
<thead>
<tr>
<th>Active Region</th>
<th>Powerlaw Slope</th>
<th>Frequency Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10848</td>
<td>2.17</td>
<td>$2.4 \times 10^{-4} - 8.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR10930</td>
<td>1.86</td>
<td>$4.3 \times 10^{-5} - 8.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR10960</td>
<td>2.00</td>
<td>$2.1 \times 10^{-4} - 5.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR10963</td>
<td>1.96</td>
<td>$1.5 \times 10^{-4} - 8.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR10978</td>
<td>2.09</td>
<td>$6.9 \times 10^{-5} - 8.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR11029</td>
<td>2.21</td>
<td>$1.3 \times 10^{-4} - 8.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR11158</td>
<td>1.88</td>
<td>$1.1 \times 10^{-4} - 8.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR11271</td>
<td>2.52</td>
<td>$1.2 \times 10^{-4} - 8.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>AR12192</td>
<td>1.91</td>
<td>$7.7 \times 10^{-5} - 8.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>AR12529</td>
<td>2.12</td>
<td>$5.1 \times 10^{-5} - 8.3 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

**Figure 3.24:** AR10978: example of a long channel power spectrum

![Image of a long channel power spectrum for AR10978](image2)

- **Spectral Power of the 1-8A X-ray Filter Timeseries of AR10978**

- **Slope in frequency range:** $[7 \times 10^{-2}, 2 \times 10^{-4}]$ $p = 0.027$
- **Slope in frequency range:** $[2 \times 10^{-4}, 1.2 \times 10^{-5}]$ $p = 0.005$
- **Slope in frequency range:** $[1.2 \times 10^{-5}, 6.2 \times 10^{-6}]$ $p = 0.002$
3.4.3 Long and Short Channel X-ray Power Spectra

Fits of the form \( S(f) \sim f^{-\beta} \) are calculated on the spectra. For all regions and both channels, 3 distinct slopes can be seen. The first slope is close to flat, while the other two are steep and vary from plot to plot. Representative plots are shown in Figures 3.24 and 3.25. The rest can be found in Appendix G. In tables 3.6 and 3.7 the fitted slopes with their corresponding ranges are shown.

<table>
<thead>
<tr>
<th>Active Region</th>
<th>( \beta_1 ) (frequency range)</th>
<th>( \beta_2 ) (frequency range)</th>
<th>( \beta_3 ) (frequency range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10848</td>
<td>0.23 (7.0 \times 10^{-7} - 2.4 \times 10^{-4})</td>
<td>2.61 (2.4 \times 10^{-4} - 8.3 \times 10^{-3})</td>
<td>- (1.4 \times 10^{-3} - 8.3 \times 10^{-3})</td>
</tr>
<tr>
<td>AR10930</td>
<td>0.39 (7.0 \times 10^{-7} - 3.0 \times 10^{-4})</td>
<td>2.93 (3.0 \times 10^{-4} - 1.4 \times 10^{-3})</td>
<td>5.58 (1.4 \times 10^{-3} - 8.3 \times 10^{-3})</td>
</tr>
<tr>
<td>AR10960</td>
<td>0.26 (8.0 \times 10^{-7} - 3.4 \times 10^{-4})</td>
<td>1.23 (3.4 \times 10^{-4} - 1.7 \times 10^{-3})</td>
<td>4.13 (1.7 \times 10^{-3} - 8.3 \times 10^{-3})</td>
</tr>
<tr>
<td>AR10963</td>
<td>0.38 (7.0 \times 10^{-7} - 4.0 \times 10^{-4})</td>
<td>2.06 (4.0 \times 10^{-4} - 1.9 \times 10^{-3})</td>
<td>3.67 (1.9 \times 10^{-3} - 8.3 \times 10^{-3})</td>
</tr>
<tr>
<td>AR10978</td>
<td>0.14 (7.0 \times 10^{-7} - 2.0 \times 10^{-4})</td>
<td>1.98 (2.0 \times 10^{-4} - 1.2 \times 10^{-3})</td>
<td>3.56 (1.2 \times 10^{-3} - 8.3 \times 10^{-3})</td>
</tr>
<tr>
<td>AR11029</td>
<td>0.48 (1.3 \times 10^{-6} - 3.9 \times 10^{-4})</td>
<td>1.94 (3.9 \times 10^{-4} - 1.3 \times 10^{-3})</td>
<td>3.45 (1.3 \times 10^{-3} - 8.3 \times 10^{-3})</td>
</tr>
<tr>
<td>AR11158</td>
<td>0.35 (1.1 \times 10^{-6} - 3.2 \times 10^{-4})</td>
<td>2.26 (3.2 \times 10^{-4} - 1.1 \times 10^{-3})</td>
<td>3.70 (1.1 \times 10^{-3} - 4.8 \times 10^{-3})</td>
</tr>
<tr>
<td>AR11271</td>
<td>0.23 (6.4 \times 10^{-7} - 1.3 \times 10^{-4})</td>
<td>2.17 (1.3 \times 10^{-4} - 8.3 \times 10^{-3})</td>
<td>-</td>
</tr>
<tr>
<td>AR12192</td>
<td>0.27 (6.4 \times 10^{-7} - 1.0 \times 10^{-4})</td>
<td>2.99 (1.0 \times 10^{-4} - 5.3 \times 10^{-3})</td>
<td>-</td>
</tr>
<tr>
<td>AR12529</td>
<td>0.87 (1.0 \times 10^{-4} - 5.0 \times 10^{-4})</td>
<td>3.97 (5.0 \times 10^{-4} - 2.0 \times 10^{-3})</td>
<td>8.00 (2.0 \times 10^{-3} - 4.5 \times 10^{-3})</td>
</tr>
</tbody>
</table>
### Table 3.7: Short Channel Flares - Power Spectra Slopes

<table>
<thead>
<tr>
<th>Active Region</th>
<th>$\beta_1$ (frequency range) (in Hz)</th>
<th>$\beta_2$ (frequency range) (in Hz)</th>
<th>$\beta_3$ (frequency range) (in Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10848</td>
<td>0.23 $(7.0 \times 10^{-7} - 4.0 \times 10^{-4})$</td>
<td>3.27 $(4.0 \times 10^{-4} - 8.3 \times 10^{-3})$</td>
<td>-</td>
</tr>
<tr>
<td>AR10930</td>
<td>0.22 $(7.0 \times 10^{-7} - 3.2 \times 10^{-4})$</td>
<td>2.86 $(3.2 \times 10^{-4} - 1.7 \times 10^{-3})$</td>
<td>5.58 $(1.7 \times 10^{-3} - 8.3 \times 10^{-3})$</td>
</tr>
<tr>
<td>AR10960</td>
<td>0.12 $(8.0 \times 10^{-7} - 3.4 \times 10^{-4})$</td>
<td>1.02 $(3.4 \times 10^{-4} - 1.9 \times 10^{-3})$</td>
<td>3.42 $(1.9 \times 10^{-3} - 8.3 \times 10^{-3})$</td>
</tr>
<tr>
<td>AR10963</td>
<td>0.24 $(7.0 \times 10^{-7} - 4.0 \times 10^{-4})$</td>
<td>1.75 $(4.0 \times 10^{-4} - 2.2 \times 10^{-3})$</td>
<td>3.96 $(2.2 \times 10^{-3} - 8.3 \times 10^{-3})$</td>
</tr>
<tr>
<td>AR10978</td>
<td>0.09 $(7.0 \times 10^{-7} - 3.0 \times 10^{-4})$</td>
<td>1.07 $(3.0 \times 10^{-4} - 1.4 \times 10^{-3})$</td>
<td>3.93 $(1.4 \times 10^{-3} - 8.3 \times 10^{-3})$</td>
</tr>
<tr>
<td>AR11029</td>
<td>0.32 $(1.3 \times 10^{-6} - 3.9 \times 10^{-4})$</td>
<td>1.81 $(3.9 \times 10^{-4} - 1.8 \times 10^{-3})$</td>
<td>3.48 $(1.8 \times 10^{-3} - 8.3 \times 10^{-3})$</td>
</tr>
<tr>
<td>AR11158</td>
<td>0.23 $(1.1 \times 10^{-6} - 3.2 \times 10^{-4})$</td>
<td>2.14 $(3.2 \times 10^{-4} - 1.4 \times 10^{-3})$</td>
<td>3.97 $(1.4 \times 10^{-3} - 4.8 \times 10^{-3})$</td>
</tr>
<tr>
<td>AR11271</td>
<td>0.06 $(6.4 \times 10^{-7} - 1.3 \times 10^{-4})$</td>
<td>1.58 $(1.3 \times 10^{-4} - 8.3 \times 10^{-3})$</td>
<td>-</td>
</tr>
<tr>
<td>AR12192</td>
<td>0.12 $(6.4 \times 10^{-7} - 1.9 \times 10^{-4})$</td>
<td>2.69 $(1.9 \times 10^{-4} - 8.3 \times 10^{-3})$</td>
<td>-</td>
</tr>
<tr>
<td>AR12529</td>
<td>0.57 $(1.0 \times 10^{-4} - 5.0 \times 10^{-4})$</td>
<td>2.91 $(5.0 \times 10^{-4} - 1.0 \times 10^{-3})$</td>
<td>10.77 $(1.0 \times 10^{-3} - 4.5 \times 10^{-3})$</td>
</tr>
</tbody>
</table>
3.5 Autocorrelation Analysis

In IDL, the built-in function A_CORRELATE computes the autocorrelation coefficients given a timeseries and values for the timelags. ACF plots are created for the following timeseries:

- the long and short channel flare timeseries
- the long and short channel background timeseries

Correlation times are visually determined for the plots. For the background, the long channel $\tau_{\text{cor}}$ are tabulated, because the short channel correlations drop below 1/e quite fast (except for AR12192). For the flares, the $\tau_{\text{cor}}$ tabulated represent both channels. Representative plots are shown in Figures 3.26, 3.27, 3.28 and 3.29.

**Table 3.8: Correlation Times**

<table>
<thead>
<tr>
<th>Active Region</th>
<th>Background (in min)</th>
<th>Flare $\tau_{\text{cor}}$ (in min)</th>
<th>Flare $\tau_{\text{cor}}$ (in min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10848</td>
<td>250</td>
<td>1/3 × 50</td>
<td></td>
</tr>
<tr>
<td>AR10930</td>
<td>125</td>
<td>1/3 × 50</td>
<td></td>
</tr>
<tr>
<td>AR10960</td>
<td>50</td>
<td>1/4 × 50</td>
<td></td>
</tr>
<tr>
<td>AR10963</td>
<td>250</td>
<td>1/3 × 50</td>
<td></td>
</tr>
<tr>
<td>AR10978</td>
<td>&gt;1000</td>
<td>1/3 × 50</td>
<td></td>
</tr>
<tr>
<td>AR11029</td>
<td>170</td>
<td>1/3 × 50</td>
<td></td>
</tr>
<tr>
<td>AR11158</td>
<td>75</td>
<td>1/3 × 50</td>
<td></td>
</tr>
<tr>
<td>AR11271</td>
<td>60</td>
<td>2/3 × 50</td>
<td></td>
</tr>
<tr>
<td>AR12192</td>
<td>100</td>
<td>4/5 × 50</td>
<td></td>
</tr>
<tr>
<td>AR12529</td>
<td>&gt;1000</td>
<td>1/3 × 50</td>
<td></td>
</tr>
</tbody>
</table>
3.5. Autocorrelation Analysis

**Figure 3.26:** AR10930: example of a smooth ACF from flare timeseries

**Figure 3.27:** AR10963: example of a fluctuating ACF from flare timeseries

**Figure 3.28:** AR11158: example of an ACF from background timeseries
3.6 Crosscorrelation Analysis

In IDL, the built-in function C_CORRELATE computes the crosscorrelation coefficients given two timeseries and timelags values. Here we are interested in the interdependencies between the flare and background timeseries. CCF plots are created for both channels. Two such plots are shown below.
3.7 Correlations Analysis

In this study, the energy $E$ of a flare is represented by the total thermal energy of each flare. The peak luminosity $P$ is represented by the peak thermal energy release rate of each flare. These, alongside the duration $T$ of each flare, are computed as described in Section 3.1.7. From the total thermal energy, the peak thermal energy release rate, and the duration of each flare, correlation plots are constructed similar to those presented in Section 1.8 - The LH Correlations Plots. Representative scatter plots are shown below for AR10930.
Chapter 3. Analysis and Results

Thermal Energy vs. Duration for AR10830

Thermal Energy vs. Peak Thermal Energy Release Rate for AR10830

slope = $2.31 \pm 0.12$

slope = $1.09 \pm 0.03$
3.7. Correlations Analysis

![Graph showing peak thermal energy release rate vs. duration for AF10830. The graph includes a linear fit line with a slope of 1.59 ± 0.14.](image_url)
Chapter 4

Discussion and Conclusions

4.1 Outline of Conclusions

- No greater than 2 powerlaws are observed at the GOES lowest energy limit of $E_{\text{min}} \sim 10^{31}$ ergs in either x-ray nor thermal energy flare distributions.

- For a given observational period, studying flares from a single active region versus studying flares from all concurrent active regions together, will not make a difference in the statistics.

- Deviation from powerlaw behavior is observed in x-ray and thermal energy flare distributions in half of the active regions studied. I propose that not all ARs can reach a true SOC state. It is the whole corona, comprised of many ARs, organizing itself over long periods of time, that is a true SOC system.

- Lack of a true SOC state could be due to:
  1. strongly driven ARs (as opposed to the standard weakly driven LH model)
  2. a SOC state has been reached in the core part of the region, but not near its boundaries.
  3. dynamics in an AR might be better represented by a locally nonconservative and homogeneously driven CA (like the continously driven OFC model)

- Flares are independent events in time

- For most ARs the driver is time-varying, alternating between one slow and one fast average driving rate. A doublewise Poisson process is usually the best description for the WTD of an AR.

- Different waiting time definitions do not affect the form of WTDs from individual ARs

- Those individual ARs that are in a true SOC state show qualitative agreement with the LH CA.

- Those individual ARs that are not in a true SOC state display SOC features in their power spectra.

- The background corona is a slowly driven SOC system, with a characteristic organization timescale of $\approx 2$ hrs.

- Individual ARs must be organizing themselves in a characteristic timescale of $\approx 12 - 40$ min.
• The background corona is operating independently of active region flaring activity.

• The whole corona, and not only individual ARs, can be in a non true SOC state for periods of \( \sim 20 \) days.

### 4.2 No greater than 2 powerlaws

The lowest observable energy for GOES XRS is \( E_{\text{min}} \simeq 10^{31} \) ergs. In the energy range of \( \approx [10^{31}, 10^{33}] \) ergs, all the x-ray peaks and thermal energy flare distributions of this study show powerlaws less than 2, within statistical error.

If a slope greater than 2 exists, it must be at flare magnitudes below the sensitivity threshold of the GOES XRS instrument (below A/B class flares). Then the energy and x-ray peaks distributions must be broken powerlaws, with a slope >2 at flare magnitudes below A/B class, and a slope <2 at flare magnitudes above B class.

Nanoflares are believed to have energy magnitudes of \( 10^{27} \) ergs and below. Thus the inability to observe >2 slopes in \([10^{31}, 10^{33}]\) ergs is not a conclusive argument against Parker’s nanoflare heating.

### 4.3 Individual Active Region Distributions vs. All Events Distributions

#### 4.3.1 AR10978, AR11158 and AR11271

AR10978, AR11158 and AR11271 (3/10) had a number of other active regions operating simultaneously. The concurrent ARs contributed some 40% of the total flare output. At least for these regions, the individual and all events distributions should show some kind of difference, if accumulating flares from many regions is to have a statistical effect.

Comparing the distributions no statistically important differences are found in the form or slope values.

#### 4.3.2 Global vs. Individual AR X-ray Peak Distributions

From Table 1.1 in the Introduction, an average slope of 1.8 can be derived for the x-ray peaks. This refers to global flare distributions. In this study, only 2/10 individual ARs show a well defined powerlaw in their x-ray peaks distribution, AR10930 and AR12192. Their average long channel slope is 1.7.

The distributions of some individual ARs could be substantially deviating in form from the scale-free powerlaws reported in the literature for large x-ray flare datasets. In general most x-ray peaks distributions in this study have a perplexed looking form. This observation agrees well with Wheatland (2010a).

This was in fact one of the reasons switched to thermal energy distributions, in the hope the peculiarities of x-ray distributions would be less exaggerated in energy distributions. Indeed this is true, yet still not all ARs show well defined powerlaw slopes even if energy distributions are studied.
4.4 Interpretation of the Thermal Energy Cum.Dist. Plots

As discussed in Section 3.2.2, thermal energy distributions are split in two categories:

- **Category 1**: The single slope active regions
  AR10930, AR10978, AR11158, AR12192 and AR12529 belong in this category.

- **Category 2**: The double slope active regions
  AR10848, AR10960, AR10963, AR11029 and AR11271 belong in this category.

In Tables 4.1 and 4.2 some phenomenologic characteristics of ARs are tabulated:

- average size of the AR.
- how the size of each region compares to the sizes of all other regions
- class of the largest flare produced by the active region.
- thermal energy range of the largest flare

### Table 4.1: Single Slope Active Regions

<table>
<thead>
<tr>
<th>Active Region</th>
<th>Largest Flare</th>
<th>Size (millionths of solar surface)</th>
<th>Comparison of Sizes</th>
<th>Range of Largest Flare (ergs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10930</td>
<td>X-class</td>
<td>482</td>
<td>3rd largest</td>
<td>$10^{35} - 10^{36}$</td>
</tr>
<tr>
<td>AR10978</td>
<td>C-class</td>
<td>205</td>
<td>3rd smallest</td>
<td>$10^{33} - 10^{34}$</td>
</tr>
<tr>
<td>AR11158</td>
<td>only 1 X-class</td>
<td>272</td>
<td>average</td>
<td>$10^{34} - 10^{35}$</td>
</tr>
<tr>
<td>AR12192</td>
<td>X-class</td>
<td>1811</td>
<td>largest</td>
<td>$10^{35} - 10^{36}$</td>
</tr>
<tr>
<td>AR12529</td>
<td>only 1 M-class</td>
<td>692</td>
<td>2nd largest</td>
<td>$10^{34} - 10^{35}$</td>
</tr>
</tbody>
</table>

### Table 4.2: Double Slope Active Regions

<table>
<thead>
<tr>
<th>Active Region</th>
<th>Largest Flare</th>
<th>Active Region Size</th>
<th>Comparison of Sizes</th>
<th>Range of Largest Flare (ergs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10848</td>
<td>C-class</td>
<td>165</td>
<td>smallest</td>
<td>$10^{33} - 10^{34}$</td>
</tr>
<tr>
<td>AR10960</td>
<td>M-class</td>
<td>288</td>
<td>average</td>
<td>$10^{34} - 10^{35}$</td>
</tr>
<tr>
<td>AR10963</td>
<td>C-class</td>
<td>277</td>
<td>average</td>
<td>$10^{33} - 10^{34}$</td>
</tr>
<tr>
<td>AR11029</td>
<td>C-class</td>
<td>224</td>
<td>average</td>
<td>$10^{33} - 10^{34}$</td>
</tr>
<tr>
<td>AR11271</td>
<td>C-class</td>
<td>197</td>
<td>2nd smallest</td>
<td>$10^{33} - 10^{34}$</td>
</tr>
</tbody>
</table>

4.4.1 Can the phenomenological differences of ARs explain the different behavior in thermal energy distributions?

Is it the size of an AR that determines its category?
Looking at Tables 4.1 and 4.2 the following conclusions can be drawn:

- large active regions tend to belong in category 1, meaning they show only 1 slope \(< -2\).
- small active regions tend to belong in category 2, meaning they show two slopes, the first \(< -2\) and the second \(> -2\).
- average sized active regions can belong to both categories, although they tend to belong more in category 2.

The tendencies outlined above are perplexed by two additional observations:

- AR10978 is a small active region, producing up to C-class flares, yet its thermal energy distribution displays a nicely defined single slope for the whole range of flares it produced. Therefore the possibility of having small active regions showing a single slope cannot be excluded.
- AR10978 and AR11029 are very similar in their phenomenologic characteristics, because they are close in size, they both produce 9 C-class flares, no M- or X-class flares, and a comparable number of B-class flares. Yet they belong in different categories, meaning that statistically they behave differently. Is there something more fundamental than the size of an active region that determines its behavior?

In what follows, I am proposing three explanations that could account for the different statistical behavior seen of category 1 and 2 ARs. I caution the reader to be mindful of the difference between the rate of the driver and the strength of the driver. The rate tells us at what average timescale similar amounts of energy are injected into the system. The strength tells us how small or large these amounts are.

### 4.4.2 First Possible Explanation

Based on the work of Charbonneau et al. (2001b), I propose that active regions with two slopes in their thermal energy cum.dist. are strongly driven. In opposition, the active regions with a single slope in their thermal energy cum.dist. are weakly driven.

A weak driver is present when \(\frac{<\delta B>}{<B>}<<1\). The energy input by the driver (the \(<\delta B>\)) is much smaller that the average energy already inside the system (the \(<B>\)). If this condition is violated the driver is strong. The LH model for example uses a weak driver.

Charbonneau et al. (2001b) investigate the effect of a strong driver on a flaring system. They stress that large clusters are a necessary condition for the existence of the SOC state. Only the avalanching of large clusters takes stress out of the system, since in small internal avalanches the field is conserved. Driving the system weakly, by adding very small increments, means that such large clusters have sufficient time to be built. The building up of such large clusters is hindered when the system is strongly driven and this in turn means that the stationary SOC state cannot be maintained for long. From their simulations they observe that strongly driven systems end up favoring mid-sized flares, as can be seen in Figure 4.1. The state strongly driven systems reach is still stationary, still avalanching, and
dissipating energy, but it is no longer a true SOC state, with clusters of all sizes and scale-free distributions.

**Figure 4.1: Avalanche Distributions for Strong Drivers**

These distributions are seen to have a first slope of small inclination and a second slope that is rather abrupt. They thus resemble the thermal energy distributions of category 2 ARs. Category 2 distributions also end up favoring mid-sized flares because the first slope is <2 and the second slope is >2.

To test the proposition of a strong driver, simulations in avalanche models with strong drivers and many different values of the fraction \( \frac{<\Delta B>}{<B>} \) have to be run. If simulated distributions are found to resemble the thermal energy distributions of category 2 ARs, then the presence of a strong driver could be well established.

### 4.4.3 Second Possible Explanation

I propose that category 2 active regions could be comprised of two separate parts:

1. an inner core area that is in a SOC state
2. an outer ring area that is in a non-SOC state

The above assumptions are based on the following observations:

1. In their article Bak et al. (1988), BTW report on a law that relates the critical exponents of the energy distribution \( \tau \), the duration distribution \( \alpha \), the fourier spectrum \( \beta \), and the exponent of avalanche growth rate \( \gamma \):

   \[
   \alpha = 2 - \beta = (\gamma + 1) \tau - 2\gamma
   \]  
   

(4.1)
This $\gamma$ is negative for the second steep slope of category 2 thermal energy distributions. The calculation of $\gamma$ is based on a scaling law that is a direct consequence of the powerlaw form of the distributions of flare parameters, no further assumptions are made. Since a negative growth exponent is unnatural, it follows that the second slope is made up of flares coming from a region that is not in a true SOC state.

2. The second part of category 2 thermal distributions resembles the tail part of the LH energy distribution for the $<\delta B> = 0$ case. In this case the system is avalanching but is not in a true SOC state (see Figure 1.2 of the Introduction).

3. The sizes of the min and max flares produced by a SOC system are related to its actual size, according to the LH simulations. Also, SOC systems have slopes $<2$. Category 2 ARs have a first slope $<2$. Thus, I consider the smallest and largest flares of this first slope as indicative of the size of a SOC-state, inner core in the AR.

4. I consider the smallest and largest flares of the second $>2$ slope of category 2 ARs as indicative of the size of a non-SOC-state, ring region in the AR.

As far as the calculation of $\gamma$ is concerned:

The duration distributions are problematic and not so reliable due to a number of observational effects and biases. To calculate the value of the exponent $\gamma$, I use the following alternative form of equation 4.1, where only the energy and fourier spectrum exponents are needed:

$$\gamma = \frac{2 - \beta - \tau}{\tau - 2}$$

The fourier power spectra exponents and the thermal energy exponents, for the category 1 and category 2 active regions, are shown in Tables 4.3 and 4.4 below.

If flares of the second slope of category 2 ARs come from regions in a SOC state, then this state would be a second SOC state. Note however that the fourier spectra of category 2 ARs show only one exponent, just like the fourier spectra of category 1 regions. Therefore this second SOC state, if it exists, despite being different should at the same time be indistinguishable from the first SOC state in terms of its fourier spectrum signature. Thus for category 2 ARs, two avalanche growth exponents $\gamma_1$ and $\gamma_2$ are calculated, one using the first slope $\alpha_{E1}$ and the fourier exponent $\beta$ and one using the second slope $\alpha_{E2}$ and the same fourier exponent $\beta$. The calculated growth exponents for category 1 ARs are shown in Table 4.5 and for category 2 ARs in 4.6.
4.4. Interpretation of the Thermal Energy Cum.Dist. Plots

Table 4.3: Single Slope Active Regions - Energy and Fourier Spectra Slopes

<table>
<thead>
<tr>
<th>Active Region</th>
<th>$\alpha_{E} = \tau$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10930</td>
<td>1.43</td>
<td>2.54</td>
</tr>
<tr>
<td>AR10978</td>
<td>1.64</td>
<td>1.31</td>
</tr>
<tr>
<td>AR11158</td>
<td>1.65</td>
<td>2.97</td>
</tr>
<tr>
<td>AR12192</td>
<td>1.41</td>
<td>1.86</td>
</tr>
<tr>
<td>AR12529</td>
<td>1.74</td>
<td>1.13</td>
</tr>
</tbody>
</table>

Table 4.4: Double Slope Active Regions - Energy and Fourier Spectra Slopes

<table>
<thead>
<tr>
<th>Active Region</th>
<th>$\alpha_{1E} = \tau_1$</th>
<th>$\alpha_{2E} = \tau_2$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10848</td>
<td>1.55</td>
<td>2.27</td>
<td>1.29</td>
</tr>
<tr>
<td>AR10960</td>
<td>1.44</td>
<td>2.38</td>
<td>1.47</td>
</tr>
<tr>
<td>AR10963</td>
<td>1.33</td>
<td>2.53</td>
<td>1.54</td>
</tr>
<tr>
<td>AR11029</td>
<td>1.49</td>
<td>2.46</td>
<td>1.93</td>
</tr>
<tr>
<td>AR11271</td>
<td>1.64</td>
<td>2.24</td>
<td>1.32</td>
</tr>
</tbody>
</table>

Table 4.5: Single Slope Active Regions - Avalanche Growth Exponents

<table>
<thead>
<tr>
<th>Active Region</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10930</td>
<td>3.46</td>
</tr>
<tr>
<td>AR10978</td>
<td>2.67</td>
</tr>
<tr>
<td>AR11158</td>
<td>7.45</td>
</tr>
<tr>
<td>AR12192</td>
<td>2.17</td>
</tr>
<tr>
<td>AR12529</td>
<td>3.35</td>
</tr>
</tbody>
</table>

Table 4.6: Double Slope Active Regions - Avalanche Growth Exponents

<table>
<thead>
<tr>
<th>Active Region</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10848</td>
<td>1.87</td>
<td>-5.70</td>
</tr>
<tr>
<td>AR10960</td>
<td>1.63</td>
<td>-4.92</td>
</tr>
<tr>
<td>AR10963</td>
<td>1.32</td>
<td>-3.89</td>
</tr>
<tr>
<td>AR11029</td>
<td>2.79</td>
<td>-5.15</td>
</tr>
<tr>
<td>AR11271</td>
<td>2.65</td>
<td>-6.50</td>
</tr>
</tbody>
</table>

4.4.4 Third Possible Explanation

Based on the work of Hamon et al. (2002), I propose that individual active regions could be better described by a continuously driven Olami-Feder-Christensen model rather than a Lu& Hamilton model. The simulations of the OFC model produce energy distributions that show both the single slope behavior that I observe in category 1 ARs and the double slope behavior I observe in category 2 ARs. The
simulated distributions predict a transition from a single powerlaw to a rather curved form, for changing values of a system parameter.

**Figure 4.2:** Energy and Duration Distributions for the continuously driven OFC model of Hamon et al. (2002) for $\nu = 10^{-5}$

![Energy Distribution](image1.png)

![Duration Distribution](image2.png)

**Figure 4.3:** Energy and Duration Distributions for the continuously driven OFC model of Hamon et al. (2002) for $\nu = 10^{-6}$

![Energy Distribution](image3.png)

![Duration Distribution](image4.png)

The continuously driven OFC model of Hamon et al. (2002) is a CA with a field defined on a 2D lattice:

- it is a bulk dissipative system. During avalanches local exchanges with neighbors are not conservative, thus the field of the lattice is not conserved.
In opposition, the LH model is conservative internally and nonconservative only at the boundaries.

- it has a finite driving rate. It becomes truly critical and scale-free in the limit of zero driving rate. For non zero driving rates, it is not a SOC model per se, but it still displays SOC features such as powerlaws with large cutoffs. Hamon et al. (2002) talk about systems at the 'edge of SOC'.

- the critical properties of the system depend on the details of the dissipation and the driving rate.

- it has a homogeneous driver. The system is evolved by adding, at each timestep, an equal amount of 'field' at all sites of the lattice. This is in opposition to the random addition of 'sand grains' of the LH model.

- it has no separation of timescales. The driving occurs at the same timescale of the relaxation.

- it is a model on a 2D lattice with open boundaries in x-direction and periodic in y-direction, in an effort to mimic the east-west, north-south anisotropy on the Sun.

- the lattice has an $L = 128$.

The total amount of field dissipated in the relaxation of one site is given by:

$$E_{dis} = (q\alpha - 1)E_i$$  \hspace{1cm} (4.3)

where $q$ : number of neighbors. If $\alpha = \frac{1}{q}$ the model is conservative. The model parameters that determine the system’s behavior are:

- the driving determined by the value of $\nu$. This $\nu$ can be thought of as the $\langle \delta B \rangle$ of the LH model. A high value for $\nu$ means that the system is strongly driven, a low value of $\nu$ means the system is weakly driven.

- the amount of conservation determined by the value of $\alpha$. For a 2D lattice, a value of $\alpha = 0.25$ means conservation. As $\alpha$ becomes smaller that 0.25, dissipation starts to exist.

**Figure 4.4:** Powerlaw slopes for the Energy Distributions of the continuously driven OFC model of Hamon et al. (2002)

<table>
<thead>
<tr>
<th>Conservation $\alpha$</th>
<th>Drive rate $\nu$</th>
<th>Exponent $\mu$</th>
<th>Cutoff $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>$1 \times 10^{-5}$</td>
<td>3.30</td>
<td>3.80</td>
</tr>
<tr>
<td>0.22</td>
<td>$1 \times 10^{-5}$</td>
<td>1.70</td>
<td>3.30</td>
</tr>
<tr>
<td>0.23</td>
<td>$1 \times 10^{-5}$</td>
<td>1.55</td>
<td>2.50</td>
</tr>
<tr>
<td>0.24</td>
<td>$1 \times 10^{-5}$</td>
<td>0.80</td>
<td>1.40</td>
</tr>
<tr>
<td>0.20</td>
<td>$1 \times 10^{-6}$</td>
<td>1.85</td>
<td>3.60</td>
</tr>
<tr>
<td>0.22</td>
<td>$1 \times 10^{-6}$</td>
<td>1.80</td>
<td>3.60</td>
</tr>
<tr>
<td>0.24</td>
<td>$1 \times 10^{-6}$</td>
<td>0.80</td>
<td>1.10</td>
</tr>
</tbody>
</table>
Figure 4.5: Powerlaw slopes for the Duration Distributions of the continuously driven OFC model of Hamon et al. (2002)

<table>
<thead>
<tr>
<th>Conservation α</th>
<th>Drive rate $\nu$</th>
<th>Exponent $\mu$</th>
<th>Cutoff $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>$1 \times 10^{-5}$</td>
<td>3.60</td>
<td>3.40</td>
</tr>
<tr>
<td>0.22</td>
<td>$1 \times 10^{-5}$</td>
<td>2.00</td>
<td>3.40</td>
</tr>
<tr>
<td>0.23</td>
<td>$1 \times 10^{-5}$</td>
<td>1.75</td>
<td>2.70</td>
</tr>
<tr>
<td>0.24</td>
<td>$1 \times 10^{-5}$</td>
<td>1.55</td>
<td>2.10</td>
</tr>
<tr>
<td>0.20</td>
<td>$1 \times 10^{-6}$</td>
<td>2.15</td>
<td>2.00</td>
</tr>
<tr>
<td>0.22</td>
<td>$1 \times 10^{-6}$</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>0.24</td>
<td>$1 \times 10^{-6}$</td>
<td>1.60</td>
<td>2.40</td>
</tr>
</tbody>
</table>

Table 4.7: Single Slope Active Regions - Powerlaw Slopes

<table>
<thead>
<tr>
<th>Active Region</th>
<th>$\alpha_E$</th>
<th>$\alpha_{T1}$</th>
<th>$\alpha_{T2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10930</td>
<td>1.43</td>
<td>1.55</td>
<td>2.38</td>
</tr>
<tr>
<td>AR10978</td>
<td>1.64</td>
<td>1.97</td>
<td>3.96</td>
</tr>
<tr>
<td>AR11158</td>
<td>1.65</td>
<td>2.21</td>
<td>4.60</td>
</tr>
<tr>
<td>AR12192</td>
<td>1.41</td>
<td>2.39</td>
<td>3.68</td>
</tr>
<tr>
<td>AR12529</td>
<td>1.74</td>
<td>2.25</td>
<td>3.53</td>
</tr>
</tbody>
</table>

Table 4.8: Double Slope Active Regions - Powerlaw Slopes

<table>
<thead>
<tr>
<th>Active Region</th>
<th>$\alpha_{E1}$</th>
<th>$\alpha_{E2}$</th>
<th>$\alpha_{T1}$</th>
<th>$\alpha_{T2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10848</td>
<td>1.55</td>
<td>2.27</td>
<td>2.22</td>
<td>3.59</td>
</tr>
<tr>
<td>AR10960</td>
<td>1.44</td>
<td>2.38</td>
<td>2.22</td>
<td>7.81</td>
</tr>
<tr>
<td>AR10963</td>
<td>1.33</td>
<td>2.53</td>
<td>2.76</td>
<td>7.00</td>
</tr>
<tr>
<td>AR11029</td>
<td>1.49</td>
<td>2.46</td>
<td>3.33</td>
<td>5.08</td>
</tr>
<tr>
<td>AR11271</td>
<td>1.64</td>
<td>2.24</td>
<td>1.68</td>
<td>2.70</td>
</tr>
</tbody>
</table>

Comparing the values of the above Tables, the following conclusions can be drawn:

- thermal energy distributions for category 2 ARs resemble the energy distributions of the OFC model for a driver with either $\nu = 10^{-5}$ or $\nu = 10^{-6}$ and conservation parameters $0.23 < \alpha < 0.24$. Both the 'hump' in mid-sized flares and the appropriate values of slope seem to be obtainable with this choice of $\alpha$.

- thermal energy distributions for category 1 ARs resemble the energy distributions of the OFC model for a driver with either $\nu = 10^{-5}$ or $\nu = 10^{-6}$ and conservation parameters $0.22 < \alpha < 0.23$. Single slope distributions with the appropriate slope values seem to be obtainable for this choice of $\alpha$.

- The duration distributions of Figures 4.2 and 4.3 are also similar to my duration distributions.
The LH model might be the suitable model for describing the sunergistic dynamics between many active regions. This is why its simulated distributions agree well with observed global flare distributions. When large numbers of flares are used to construct distributions, they come from a large number of active regions that appeared on the Sun at different times. Such distributions show all the features of a true SOC state because they express the dynamics of the whole magnetic field network of the corona, which is conservative and on the whole driven by the random photospheric motions of the photosphere. Therefore the magnetic field self organizes critically spatially on the whole surface of the Sun and on timescales much larger than the lifetime of individual active regions.

Yet, not every single AR has to be in a true SOC state. It could be in a state ‘at the edge of SOC’ as those seen in the OFC simulations. In such states some SOC features still exist but the symmetry of scale-free behavior over all scales is broken. When concentrated amounts of magnetic field emerge from the convection zone, the driving resembles more the homogeneous driving of the OFC model. Moreover, the field in the region doesn’t need to be conserved locally, it can be dissipated to other parts of the magnetic network. If magnetic field emergence loads substantial amounts of field onto an AR, the driver is strong, and oppositely, if it loads small amounts, the driver is weak. Yet the tuning of the parameter of conservation $\alpha$ is what determines the evolution of the system. If $0.22 < \alpha < 0.23$, the system will display scale-free features more clearly (Category 1 ARs). If $0.23 < \alpha < 0.24$, the system will display two slopes separated by a ‘hump’ (Category 2 ARs). Note that for Category 2 ARs $\alpha$ is closer to 0.25, yet deviation from powerlaw is more prominent.

### 4.5 Waiting Times

#### 4.5.1 Waiting Time Correlations

Scatter plots for the waiting time vs. the energy of the previous/following flare were constructed. No correlation between the quantities was found. Thus, flares in individual ARs are independent from one another in time. The system will produce the next flare ‘not remembering’ what flare it created just before, and ‘not thinking’ of what kind of flare it will produce afterwards.

#### 4.5.2 WTDs in log-lin axis

8 of the active regions studied have two average waiting times, while 2 active regions have a single average waiting time. These conclusions agree with the findings of Wheatland (2001). A number of conclusions can be drawn:

- flares are independent events in time since they can be described by Poissonian distributions.

- during its lifetime, an active region isn’t necessarily driven at a constant average rate. The driver is therefore ‘feeding’ energy into the region with different rates at different times, and it must be doing so in an alternating manner, switching from one rate to the other. I am claiming this, because the sequence of waiting times of events is not comprised of, for example, events with a shorter waiting times in the beginning and after some time only events with longer waiting times. The short and long waiting times are
mixed, thus the driver must be alternating between slow and fast driving rates during the lifetime of an active region.

- if we want to characterize the driver of active regions (the turbulent convective motions of photospheric plasma) on the average, we would say that as far as its rate is concerned, it is a time-varying driver and its variations, although substantial, are concentrated around a low and a high average rate.

4.5.3 Does event definition affect the WTDs?

As discussed in Section 1.9.4 of the Introduction, the method of event definition has been blamed for creating different types of WTDs given the same dataset. To test this for the 10 ARs studied, I created different waiting time distributions using the different definitions mentioned in 1.9.4. No substantial difference in the form of the WTDs was found.

4.5.4 Is there a relation between double slopes in thermal energy distributions and in WTDs?

All category 2 active regions have two slopes in their thermal energy distributions and two slopes in their WTDs. The following question could be raised: Do all flares from the each slope of the energy distribution correspond to only one of the average two waiting times?

A special type of plots was created to address this. On the WTDs of the category 2 active regions (log-lin axis), the waiting times of the flares of the second energy slope were overplotted. No connection was found.

4.5.5 Waiting Time Model Comparisons

For most (6/10) of the ARs analyzed in this study the Poisson distribution (single or double according to the region) is the best fit for the WTD. Therefore, generally we would expect the WTD of an AR to be Poissonian. But, for the tail part of some (4/10) of the regions analyzed, Wheatland’s fit is better than Poisson’s.

This implies that active regions with long timescale correlations can exist (a powerlaw suggests system-wide synergistic dynamics). This correlation in the large waiting times is not a correlation in the sense of relation of one event to the next. It is a correlation in a SOC sense, meaning that the system ‘decides’ it will create events with large waiting times for which there will be no preferred average waiting time. This is in contrast to the initial part of the WTD, which implies that a preferred scale for the short waiting times exists.

The lack of an average waiting time at large timescales implies the lack of an average driving rate at large timescales. Could it be then that for some active regions, when looking at long timescales, the driver has scale free properties in the way it is injecting energy into the system? In this case this long timescale scale-free driving is mapped onto the response of the active region in the form of scale-free long waiting times.

Looking at the form of WTDs and their respective drivers from Figure 3.17, my observations seem to suggest that:

- for short timescales the driver of active regions is step-like, alternating between an average high and an average low rate
• for long timescales the driver of the solar coronal magnetic field is pulse
like, following a trend reminiscent of solar cycles.

The proposed reasoning is that these two observations can be reconciled if the
step-like features are thought of as unresolved fine details of a general pulse like
driving rate. According to the timescale of observation then, the driver will ap-
pear different. If we ‘zoom in’ and look at the flaring from an active region, the
step-like features are visible. If we ‘zoom out’ and observe the evolution of flaring
in solar cycle timescales, the pulse-like features are visible.

4.6 Comparisons with the LH model distributions

Category 1 ARs show well defined powerlaw slopes in their thermal energy dis-
tributions and can therefore be considered true SOC systems. In what follows, I
compare these with the LH distributions.

As far as the energy distributions are concerned:

The powerlaw slopes for the thermal energy distributions and the duration
distributions of category 1 regions are shown in Table 4.7. The powerlaw slopes
for the peak energy release rate distributions are shown in Table 4.9 below.

<table>
<thead>
<tr>
<th>Active Region</th>
<th>$\alpha_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10930</td>
<td>1.48</td>
</tr>
<tr>
<td>AR10978</td>
<td>1.82</td>
</tr>
<tr>
<td>AR11158</td>
<td>1.77</td>
</tr>
<tr>
<td>AR12192</td>
<td>1.56</td>
</tr>
<tr>
<td>AR12529</td>
<td>1.76</td>
</tr>
</tbody>
</table>

The averages of the $\alpha_E$, $\alpha_P$, and $\alpha_T$ values of the LH simulations as seen in
Figure 1.3 in the Introduction are:

\[
\alpha_{E_{LH}} = 1.47 \approx 1.5 \quad (4.4)
\]

\[
\alpha_{P_{LH}} = 1.72 \approx 1.7 \quad (4.5)
\]

\[
\alpha_{T_{LH}} = 1.56 \approx 1.6 \quad (4.6)
\]

These values represent averages from a number of active regions of different
sizes.
The averages of the $\alpha_E$, $\alpha_P$, and $\alpha_T$ values of the category 1 active regions are:

\[
\begin{align*}
\alpha_{E_{\text{obs}}} &= 1.57 \pm 1.6 \quad (4.8) \\
\alpha_{P_{\text{obs}}} &= 1.68 \pm 1.7 \quad (4.9) \\
\alpha_{T_{\text{obs}1}} &= 2.85 \pm 2.9 \quad (4.10) \\
\alpha_{T_{\text{obs}2}} &= 2.07 \pm 2.1 \quad (4.11)
\end{align*}
\]

These values also represent averages from a number of active regions of different sizes. Therefore it is reasonable to compare the simulated to the observed averages.

The duration distributions are generally the hardest to fit, because they have irregular curves that sometimes show three different slopes. The $\alpha_{T_{\text{obs}1}}$ and $\alpha_{T_{\text{obs}2}}$ values above are calculated in two different ways:

- the average of the two $\alpha_{T1}$ and $\alpha_{T2}$ values of Table 4.7 are calculated, separately for each category 1 active region. Then these averages are again averaged for the calculation of $\alpha_{T_{\text{obs}1}}$.
- the average of all the $\alpha_{T1}$ values for the category 1 active regions is calculated. This is the $\alpha_{T_{\text{obs}2}}$ value.

The agreement of the observed $\alpha_E$ and $\alpha_P$ to the simulated ones is remarkable. Yet the duration powerlaws are not in agreement, which is to be expected (see Introduction).

The agreement between observations and simulations would not be so striking had we used the x-ray peaks distributions for the comparison. Nevertheless, average slopes from the x-ray peaks distributions are calculated for category 1 ARs and are shown in Table 4.10 below.

**Table 4.10: Category 1 Active Regions - X-ray Peaks Slopes**

<table>
<thead>
<tr>
<th>Active Region</th>
<th>LongChannelSlope</th>
<th>ShortChannelSlope</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR10930</td>
<td>1.57</td>
<td>1.47</td>
</tr>
<tr>
<td>AR10978</td>
<td>1.82</td>
<td>1.69</td>
</tr>
<tr>
<td>AR11158</td>
<td>1.93</td>
<td>1.73</td>
</tr>
<tr>
<td>AR12192</td>
<td>1.74</td>
<td>1.62</td>
</tr>
<tr>
<td>AR12529</td>
<td>2.16</td>
<td>1.93</td>
</tr>
</tbody>
</table>

The average slope for the long channel is $1.84 \approx 1.8$ and for the short channel $1.69 \approx 1.7$. Despite the fact then that the x-ray peaks distributions do not show well defined powerlaws, taking an average value for the slope for each active region and then averaging again between all regions seems to be a good method to recover powerlaw slopes close to the simulated ones.

As far as flare parameter correlations are concerned:
Averaging between the powerlaw fits of the correlation plots of category 1 active regions, the following relations are obtained:

\[ E \propto P^{1.12} \]  
\[ E \propto T^{1.96} \]  
\[ P \propto T^{1.19} \]

(4.13)  
(4.14)  
(4.15)

Compare these with the correlation relations of LH:

\[ E \propto P^{1.82} \]  
\[ E \propto T^{1.77} \]  
\[ P \propto T^{0.90} \]

(4.17)  
(4.18)  
(4.19)

(4.20)

For the \( E - T \) and \( P - T \) relations there is relative agreement, even though in the \( P \) vs. \( T \) correlation points are quite scattered. For the \( E - P \) relation though the agreement is not at all satisfying, not even if we take into account the sigma values of the calculated fits (\( \bar{\sigma} = 0.05 \implies 1.12 + 0.05 = 1.17 \) still much than 1.82).

### 4.7 Fourier Plots

As far as the energy release rate spectra are concerned:

According to the second possible explanation explained in Section 4.4.3, the inner area of an active region is in a well defined SOC state, whereas the outer ring area is in a non-SOC state. Using this reasoning I claim that the powerlaws seen in the energy fourier spectra are signatures of energy release from the inner SOC area of an AR. Averaging the \( \beta \) values for all 10 active regions I obtain:

\[ \beta = 1.7 \approx 2 \]

(4.21)

in agreement with the simulations of LH, who predict \( \beta \approx 2 \). This agreement leads to the conclusion that active regions display SOC features in their behavior, irrespective of the strength of the driver, the extent of their area being in a SOC state, or whether they are in a true SOC state or at ‘the edge of SOC’. All active regions then, have dynamics that can successfully be described in an abstract manner by simple local next-neighbor interactions, coupled with the existence of a driver loading energy on the system.

As far as the x-ray spectra are concerned:

For the long channel x-ray spectra:

- Taking the average of the \( \beta_1 \) values yields: \( \bar{\beta}_1 = 0.4 \)
- Taking the average of the \( \beta_2 \) values yields: \( \bar{\beta}_2 = 2.1 \)
Chapter 4. Discussion and Conclusions

- Taking the average of the $\beta_3$ values yields: $\bar{\beta}_3 = 4.0$
- The break frequency in the transition from $\beta_1$ to $\beta_2$ averages to: $2.9 \times 10^{-4}\, Hz$
- The break frequency in the transition from $\beta_1$ to $\beta_2$ averages to: $1.5 \times 10^{-3}\, Hz$

For the short channel x-ray spectra:

- Taking the average of the $\beta_1$ values yields: $\bar{\beta}_1 = 0.2$
- Taking the average of the $\beta_2$ values yields: $\bar{\beta}_2 = 1.9$
- Taking the average of the $\beta_3$ values yields: $\bar{\beta}_3 = 3.9$
- The break frequency in the transition from $\beta_1$ to $\beta_2$ averages to: $3.2 \times 10^{-4}\, Hz$
- The break frequency in the transition from $\beta_1$ to $\beta_2$ averages to: $1.6 \times 10^{-3}\, Hz$

For the x-ray spectra of both channels then, the following conclusions can be drawn:

- There is a first flat part in the x-ray spectra of solar flares with an exponent $\beta_1 \approx 0$
- There is a second powerlaw part in the x-ray spectra of solar flares with an exponent $\beta_1 \approx 2$
- There is a third powerlaw part in the x-ray spectra of solar flares with an exponent $\beta_1 \approx 4$
- The break frequency between the first and the second part is close to $1 \times 10^{-4}\, Hz$
- The break frequency between the second and the third part is close to $1 \times 10^{-3}\, Hz$

There are two articles in the literature which are relevant to the analysis in this study. The findings of UeNo et al. (1997) disagree with this study, while the findings of Ireland et al. (2014) agree.

As far as the background spectra are concerned:

All powerlaw slopes of the background spectra are remarkably close to 2, except maybe for the background spectrum of AR11271 which has a slope of 2.52. The average value of the slopes is:

$$\beta_{bg} = 2.1 \approx 2$$  \hspace{1cm} (4.22)

To my knowledge this is the first time power spectra of x-ray background timeseries have been studied. I investigated them because the ACF plots of the background timeseries revealed long time correlations (longer than the flare time correlations) for all ARs. The existence of long spatiotemporal correlations in a system suggests the system operates in a similar way in all length and timescales,
it is a SOC system. Could it be that the background corona is a SOC system in its own right? The necessary long timescale correlations and powerlaw form $S(f) \sim f^{-\beta}$, $0 < \beta < 2$ in the background spectra are seen to exist.

As discussed in the review article of Klimchuk et al. (2006), it is suggested by many researchers nowadays that the diffuse corona is heated by nanoflares that come at an intermediate to high frequency of occurrence. Such ‘trains’ of nanoflares appear as a steady background, because their high rate of occurrence doesn’t allow for significant temperature variations. As a consequence, the radiation produced by such a background will also fluctuate insignificantly. Of course for such a scenario to be true, footpoint driving must be replenishing the magnetic energy lost in real time, meaning that driving motions faster and smaller in scale than those currently observed in the photosphere must be present. Nevertheless, my observations of long timerange correlations and powerlaw fourier spectra in the background is in agreement with a scenario of high rate nanoflares heating the diffuse corona.

### 4.8 Autocorrelation Plots

The correlation times for the flare timeseries quickly drop below 1/e, suggesting that the flaring process has short term memory, in contrast to SOC theory. Since all other evidence is in support of active regions being at least in some area of their extent in a SOC state or at least in a state that resembles a SOC state, how are we to interpret this lack of long time correlations in the flare timeseries? I believe the answer is that for the flaring mechanism, a correlation time of 12 - 40 min is in fact long. If one thinks in terms of the reconnection duration, and if this reconnection duration is much less than 1 min, then 12-40 min is a long correlation time. Therefore the observations of this study for the correlation time of flare timeseries must be interpreted as indicative of the very short duration of reconnection, setting a limit for MHD theory and simulations.

As far as the background correlation times are concerned, an average $t_{\text{cor}} \approx 2$ hrs can be derived from observations. This average background correlation time is calculated from all ARs excluding AR10978 and AR12529, because for these two regions the long channel background stays above 1/e even after 1000 min. These long time correlations suggest that the background corona is a system of long term memory. This observation, coupled with the $S(f) \sim f^{-2}$ form of the background power spectra, suggests the background corona operates as a SOC system. The main difference between the background corona as a SOC system and active regions as SOC systems, seems to be the length of time these system need to express their SOC dynamics.

It might be interesting to note that, on average, the behavior below 1/e of ACF plots of category 1 ARs is smoother than the behavior below 1/e of category 2 regions. The ACF plots of category 2 regions are seen to be fluctuating significantly, with several spikes which are although always below 1/e. Oppositely, most ACF plots of category 1 regions are nearly zero slope lines after they drop below 1/e. Given the fact that category 2 regions are not in a clearly defined SOC state, could it then be that these fluctuations are a signature of the lack of a true SOC state?
4.9 Crosscorrelation Plots

The crosscorrelation plots are created in order to investigate the degree of interdependence between the flare and the background timeseries:

- If long correlation times are seen in these plots, then the background must be operating in response to the activity of the flaring mechanism.
- If short correlation times or no correlation is seen in these plots, then the background is operating independent of the flaring mechanism.

No correlation between the flare and background timeseries was found. From 0 - 1000 min the CCF is below $1/e$ from the beginnig til the end of the plot, with only a limited number of spikes reaching the level of $1/e$ for one or two active regions. Even for timeperiods equal to typical flare durations (several minutes), correlation of flares to background is not observed. This observation is in agreement to previous statements about the background corona being a SOC system in its own right. Could it be that to solve the coronal heating problem we should focus more on the diffuse corona?

4.10 Lack of a true SOC state: only seen in individual active regions?

To answer this question, a time period of solar activity during which flares originated from many active regions was searched. Looking at the LMSAL Archive, such a time period was found for 2016/4/24 - 2016/5/14 (21 days). Eight active regions were flare productive: AR12532, AR12535, AR12536, AR12537, AR12539, AR12540, AR12541, and AR12542. They produced 102 flares: 14 C-class flares and 88 flares B-class flares. This list of events was analyzed using the algorithms and procedures described in Analysis. In the end, the same plots as for all other regions were created.

The thermal energy distribution for this ‘mix’ of ARs has two distinct parts, a first part of powerlaw $<2$ and a second part of powerlaw $>2$, just like category 2 ARs. Also all other plots and distributions are comparable to those of category 2 ARs. Maybe we didn’t even need to construct the plots to conjecture that, for these 21 days, the corona was not in a SOC state. There are no M- and X-class flares, and thus how could it be that the system was scale-free during this time?

I suggest the following:

- The solar corona might or might not be in a SOC state when observed for periods of time of the scale of months. Based on the all events distributions of this study, it seems that half of the time the corona manages to self organize in a time period of one month. In the other half of the time it doesn’t manage to do that, and needs longer time periods to reach a true SOC state. During these times SOC-like features are present in the corona and in individual ARs, but the system hasn’t reached the stationary state yet, where complete scale invariance is seen to exist.

- This observation is reminiscent of the number of iterations that are needed in a SOC CA for the grid to reach the statistically stationary SOC state. It seems that the coronal magnetic field needs a large number of ‘iterations’,
meaning long periods of time, to reach a true SOC state. Before reaching this, it is still avalanching and its timeseries resemble in structure those of the SOC state, but it cannot produce flares of all sizes.

- Maybe the only difference between category 1 and category 2 ARs is that we 'sample' the corona while it has reached a true SOC state or not, respectively.
Appendix A

The Analysis Algorithm in IDL
pro ar10848

;----------------------------------------------------------------------
; Written by : Lily Kromyda
; Last modified : 2016
;----------------------------------------------------------------------

; CHANGE string number
; CHANGE days
; CHANGE goes_meas=ar_1****
; THEN THE ANALYSIS IS DONE AUTOMATICALLY CREATING RESULT PLOTS IN THE FOLDER WHERE
ar_1**** IS LOCATED

number='10848'

give number of days of the timeseries in variable days
days=16UL

; activate save file ar_1****_t_em.sav containing the temperature and emission measure for the flare of AR1****
; these are calculated with the SSWIDL procedure goes_chianti_tem (found in sswold/gen/idl/synoptic/goes/)
; in an SSWIDL command line session and saved in a structure in this .sav file
; goes_chianti_tem, sxr_flare, hxr_flare, t, em, satellite=...
restore, '/home/gkromyda/IDLWorkspace80/Default/AR'+number+'/ar_'+number'_t_em.sav'

; copy ar_1**** structure into goes_meas


; filepath for the flare events .txt file
file = filepath(number+'_flares_lmsal.txt', root_dir='/home',
subdir=['gkromyda','IDLWorkspace80','Default','AR'+number])

;

;----------------------------------------------------------------------
;----------------------------------------------------------------------

s : number of data points in the timeseries
size = size(goes_meas.yclean[*,0])
s = size[1]

; do you have a 3-sec GOES timeseries of a 2-sec(mix with some 3-sec) GOES timeseries ?
; how many points do you lose if your timeseries is 3-sec ?
expeected_3sec_pts=days*86400/3
lost_3sec=expected_3sec_pts-s
; how many points do you lose if your timeseries is 2-sec ?
expeected_2sec_pts=days*86400/2
lost_2sec=expected_2sec_pts-s

if (lost_3sec eq 0) then type_of_ts='3-sec complete'
if (lost_2sec eq 0) then type_of_ts='2-sec complete'
if (lost_3sec ne 0) and (lost_2sec ne 0) then begin
if (lost_3sec lt lost_2sec) then type_of_ts='3-sec incomplete' else type_of_ts='2-sec incomplete'
endif

if (type_of_ts eq '3-sec complete') then begin
newtarray=goes_meas.tarray
newyclean0=goes_meas.yclean[*,0]
newyclean1 = goes_meas.yclean[* ,1]
; bin with average for 1-min data points
; the 3-sec timeseries is contained in the fluxs (soft) fluxh (hard) arrays
num_minutes = s / 20
fluxs = fltarr(num_minutes)
fluxh = fltarr(num_minutes)
j = 0UL
for i = 0UL, s - 20, 20 do begin
    fluxs[j] = mean(newyclean0[i : i + 19])
    fluxh[j] = mean(newyclean1[i : i + 19])
    j++
endfor
endif

if (type_of_ts eq '2-sec complete') then begin
    newtarray = goes_meas.tarray
    newyclean0 = goes_meas.yclean[* ,0]
    newyclean1 = goes_meas.yclean[* ,1]
    ; bin with average for 1-min data points
    ; the 3-sec timeseries is contained in the fluxs (soft) fluxh (hard) arrays
    num_minutes = s / 30
    fluxs = fltarr(num_minutes)
    fluxh = fltarr(num_minutes)
j = 0UL
for i = 0UL, s - 30, 30 do begin
    fluxs[j] = mean(newyclean0[i : i + 29])
    fluxh[j] = mean(newyclean1[i : i + 29])
    j++
endfor
endif

if (type_of_ts eq '3-sec incomplete') then begin
    ; go through the tarray (contains second of observation) and find positions where jumps exist.
    ; normally every point should be time of point before + 3sec
    ; if that's not the case save the position in gap_pos array
    ; add as many 3sec data points as needed to fill the gap
j = 0U
gap_pos = ulonarr(1000)
added_pts = ulonarr(1000)
for i = 0UL, s - 2 do begin
    jump = goes_meas.tarray[i + 1] - goes_meas.tarray[i]
    if (jump gt 3UL) then begin
        gap_pos[j] = i
        print, 'jump at', gap_pos[j], 'is', jump, ' sec'
        added_pts[j] = jump / 3 - 1
        j++
    endif
endfor
gap_pos = gap_pos[0 : j - 1]
added_pts = added_pts[0 : j - 1]

; create the corrected soft and hard flux and time arrays adding as many more points as needed
newyclean0 = fltarr(s + total(added_pts))
newyclean1 = fltarr(s + total(added_pts))
newtarray = lonarr(s + total(added_pts))

;--------------------------------------------------------------------------------------------
; before 1st gap fill new arrays with old values
; but round the times to get rid of the decimals
newyclean0[0 : gap_pos[0]] = goes_meas.yclean[0 : gap_pos[0], 0]
newyclean1[0 : gap_pos[0]] = goes_meas.yclean[0 : gap_pos[0], 1]
newtarray[0 : gap_pos[0]] = goes_meas.tarray[0 : gap_pos[0]]
; fill in the missing points with values that are dy/added_pts+1=dy/#spaces apart
; remember! in the new arrays you need to take into account the number of points you are adding to place values correctly -> you need tot_added_pts
; j -> number of groups of gaps
tot_added_pts=0U
for i=0UL,j-1 do begin
  diff0=goes_meas.yclean[gap_pos[i]+1,0]-goes_meas.yclean[gap_pos[i],0]
  dy0=diff0/(added_pts[i]+1)
  diff1=goes_meas.yclean[gap_pos[i]+1,1]-goes_meas.yclean[gap_pos[i],1]
  dy1=diff1/(added_pts[i]+1)
  for k=1U,added_pts[i] do begin
    newyclean0[gap_pos[i]+tot_added_pts+k]=newyclean0[gap_pos[i]+tot_added_pts+k-1]+dy0
    newyclean1[gap_pos[i]+tot_added_pts+k]=newyclean1[gap_pos[i]+tot_added_pts+k-1]+dy1
  endfor
  if (i ne j-1) then begin
    newyclean0[gap_pos[i]+tot_added_pts+1:gap_pos[i+1]+tot_added_pts]=goes_meas.yclean[gap_pos[i]+1:gap_pos[i+1],0]
  endif else begin
    newyclean0[gap_pos[i]+tot_added_pts+1:s-1+tot_added_pts]=goes_meas.yclean[gap_pos[i]+1:s-1,0]
  endif
endfor
; bin with average for 1-min data points
; the 3-sec timeseries is contained in the fluxs (soft) fluxh (hard) arrays
num_minutes=s/20
fluxs=fltarr(num_minutes)
fluxh=fltarr(num_minutes)
j=0UL
for i=0UL,s-20,20 do begin
  fluxs[j]=mean(newyclean0[i:i+19])
  fluxh[j]=mean(newyclean1[i:i+19])
  j++
endfor
endif

; if (type_of_ts eq '2-sec incomplete') then begin
; go through the tarray (contains second of observation) and find positions where jumps exist.
; first round the observation times b/c they are given in decimal form
; normally every point should be time of point before + 2 sec or + 3 sec
; if that's not the case save the position in gap_pos array
; add points following the rule -> seconds missing/2 if the division has mod=0 or seconds missing-3/2 if the division
seconds missing/2 has mod not 0
; e.g. 10 sec missing -> 10/2=5 -> 4 points aka 5 spaces of 2 sec added
; e.g. 35 sec missing -> 35/2=17 -> 17 points aka 16 spaces of 2 sec added & 1 point aka 1 space of 3 sec
j=0UL
k=0UL
l=0UL
gap_pos=ulonarr(1000)
added_pts=uintarr(1000)
fill_method=strarr(1000)
for i=0UL,s-2 do begin
  jump=round(goes_meas.tarray[i+1])-round(goes_meas.tarray[i])
  if (jump gt 3L) then begin
    gap_pos[j]=i
    print, 'jump at', gap_pos[j], ' is', jump, ' sec'
    if (jump mod 2L) eq 0L then begin
      added_pts[j]=(jump/2L)-1
      fill_method[j]='all 2sec'
    endif else begin
      added_pts[j]=(jump-3L)/2L
      fill_method[j]='1 3sec + others 2sec'
    endelse
    j++
  endif
  if (jump eq 2L) then begin
    regular_2sec[k]=i
    k++
  endif
  if (jump eq 3L) then begin
    regular_3sec[l]=i
    l++
  endif
endfor

gap_pos=gap_pos[0:j-1]
added_pts=added_pts[0:j-1]
fill_method=fill_method[0:j]
regular_2sec=regular_2sec[0:k-1]
regular_3sec=regular_3sec[0:l-1]

; create the corrected soft and hard flux and time arrays adding as many more points as needed
newyclean0=fltarr(s+total(added_pts))
newyclean1=fltarr(s+total(added_pts))
newtarray=lonarr(s+total(added_pts))

; before 1st gap fill new arrays with old values
; but round the times to get rid of the decimals
newyclean0[0:gap_pos[0]]=goes_meas.yclean[0:gap_pos[0],0]
newyclean1[0:gap_pos[0]]=goes_meas.yclean[0:gap_pos[0],1]
newtarray[0:gap_pos[0]]=round(goes_meas.tarray[0:gap_pos[0]])

; fill in the missing points with values that are dy/added_pts+1=dy/#spaces apart
; remember! in the new arrays you need to take into account the number of points you are adding to place values correctly -> you need tot_added_pts
; j -> number of groups of gaps
tot_added_pts=0U
for i=0UL,j-1 do begin
  if fill_method[i] eq 'all 2sec' then begin
    ; correcting the 2sec gaps...
    diff0=goes_meas.yclean[gap_pos[i]+1,0]-goes_meas.yclean[gap_pos[i],0]
    dy0=diff0/(added_pts[i]+1)
    diff1=goes_meas.yclean[gap_pos[i]+1,1]-goes_meas.yclean[gap_pos[i],1]
    dy1=diff1/(added_pts[i]+1)
    newyclean0[gap_pos[i]+tot_added_pts+1]=goes_meas.yclean[gap_pos[i],0]+dy0
    newyclean1[gap_pos[i]+tot_added_pts+1]=goes_meas.yclean[gap_pos[i],1]+dy1
    newtarray[gap_pos[i]+tot_added_pts+1]=round(goes_meas.tarray[gap_pos[i]]+2L)
  endfor
  for k=2U,added_pts[i] do begin
    newyclean0[gap_pos[i]+tot_added_pts+k]=newyclean0[gap_pos[i]+tot_added_pts+k-1]+dy0
newyclean1[gap_pos[i]+tot_added_pts+k]=newyclean1[gap_pos[i]+tot_added_pts+k-1]+dy1
newarray[gap_pos[i]+tot_added_pts+k]=newarray[gap_pos[i]+tot_added_pts+k-1]+2L.
endfor

; tot_added_pts : contains number of all added points from the gaps already corrected
tot_added_pts=tot_added_pts+added_pts[i]
print, 'tot_added_pts at',i+1,'gap', tot_added_pts
endif else begin
; correcting the 1 3sec + all others 2sec gaps...
diff0=goes_meas.yclean[gap_pos[i]+1,0]-goes_meas.yclean[gap_pos[i],0]
dy0=diff0/(added_pts[i]+1)
diff1=goes_meas.yclean[gap_pos[i]+1,1]-goes_meas.yclean[gap_pos[i],1]
dy1=diff1/(added_pts[i]+1)
newyclean0[gap_pos[i]+tot_added_pts+1]=goes_meas.yclean[gap_pos[i],0]+dy0
newyclean1[gap_pos[i]+tot_added_pts+1]=goes_meas.yclean[gap_pos[i],1]+dy1
newarray[gap_pos[i]+tot_added_pts+1]=round(goes_meas.tarray[gap_pos[i]])+3L
for k=2U,added_pts[i] do begin
  newyclean0[gap_pos[i]+tot_added_pts+k]=newyclean0[gap_pos[i]+tot_added_pts+k-1]+dy0
  newyclean1[gap_pos[i]+tot_added_pts+k]=newyclean1[gap_pos[i]+tot_added_pts+k-1]+dy1
  newarray[gap_pos[i]+tot_added_pts+k]=newarray[gap_pos[i]+tot_added_pts+k-1]+2L.
endfor

; tot_added_pts : contains number of all added points from the gaps already corrected
for k=2U,added_pts[i] do begin
  newyclean0[gap_pos[i]+tot_added_pts+k]=newyclean0[gap_pos[i]+tot_added_pts+k-1]+dy0
  newyclean1[gap_pos[i]+tot_added_pts+k]=newyclean1[gap_pos[i]+tot_added_pts+k-1]+dy1
  newarray[gap_pos[i]+tot_added_pts+k]=newarray[gap_pos[i]+tot_added_pts+k-1]+2L.
endfor

;-------------------------------------------------------------------------------------------------------------------
; Create the 1-min timeseries
; Problem! the existance of non equidistant data points in time e.g. you don't have exactly 60,120,180,240 etc..
; find the positions in tarray where 59,60 or 61 sec exist, 119,120 or 121 sec exist etc... and average to the closest point
to a multiple of 60
j=0UL
avrg_pos=ulonarr(s)
for i=0UL,s-1+total(added_pts) do begin
  if ((newtarray[i] eq 60*(j+1)) or (newtarray[i] eq 60*(j+1)+1) or (newtarray[i] eq 60*(j+1)-1)) then begin
    avrg_pos[j]=i
    j++
  endif
endfor

; 1-min bin the timeseries by averaging the flux values between the avrg_pos positions
size=size(avrg_pos)
num_minutes=size[1]
fluxs=fltarr(num_minutes)
fluxh=fltarr(num_minutes)
for i=1,num_minutes-1 do begin
  fluxs[i]=mean(newyclean0[avrg_pos[i-1]:avrg_pos[i]])
  fluxh[i]=mean(newyclean1[avrg_pos[i-1]:avrg_pos[i]])
read into IDL the 1****_flares_lmsal.txt file containing the information for the AR's flares
this information goes into an array of structures named data of type string
the flares array of structures splits the information of each flare in the fields date, start, stop, peak, size, loc, ar in type string

nrows = file_lines(file)
openr, lun, file, /get_lun
data = strarr(nrows)
readf, lun, data
free_lun, lun
structure = { date:'', start:'', stop:'', peak:'', size:'', loc:'', ar:'' }
flares = replicate(structure, nrows)
FOR i=0B,nrows-1 DO BEGIN
  result = strsplit(data[i],'
' + STRING(9B) + '
'+ , /extract)
  flares[i].date=result[0]
  flares[i].start=result[1]
  flares[i].stop=result[2]
  flares[i].peak=result[3]
  flares[i].size=result[4]
  flares[i].loc=result[5]
  flares[i].ar=result[6]
ENDFOR

the information contained in flares string array of structures is converted to numerical data and placed in the data array of structures

structure = { day:0UL, ts_hrs:0UL, ts_min:0UL, tp_hrs:0UL, tp_min:0UL, te_hrs:0UL, te_min:0UL }
data = replicate(structure, nrows)
peak_flux = fltarr(nrows)
FOR i=0B,nrows-1 DO BEGIN
  result = strsplit(flares[i].date,"/",/extract)
data[i].day = ulong(result[2])
  result = strsplit(flares[i].start,':',/extract)
data[i].ts_hrs = ulong(result[0])
data[i].ts_min = ulong(result[1])
  result = strsplit(flares[i].stop,':',/extract)
data[i].te_hrs = ulong(result[0])
data[i].te_min = ulong(result[1])
  result = strsplit(flares[i].peak,':',/extract)
data[i].tp_hrs = ulong(result[0])
data[i].tp_min = ulong(result[1])
class = strmid(flares[i].size,0,1)
mag = float(strmid(flares[i].size,1,3))
if class eq 'A' then peak_flux[i]=mag*10^(-8.0)
if class eq 'B' then peak_flux[i]=mag*10^(-7.0)
if class eq 'C' then peak_flux[i]=mag*10^(-6.0)
if class eq 'M' then peak_flux[i]=mag*10^(-5.0)
if class eq 'X' then peak_flux[i]=mag*10^(-4.0)
ENDFOR

convert start, peak, end times in minutes
ts = ulonarr(nrows)
tp = ulonarr(nrows)
te = ulonarr(nrows)
flag=0B
for i=0B,nrows-1 do begin
  if (data[i].day ge data[0].day) then begin
for days in the same month
ts[i] = (data[i].day - data[0].day) * 1440 + data[i].ts_hrs * 60 + data[i].ts_min
tp[i] = (data[i].day - data[0].day) * 1440 + data[i].tp_hrs * 60 + data[i].tp_min
te[i] = (data[i].day - data[0].day) * 1440 + data[i].te_hrs * 60 + data[i].te_min

; for flares with start b/f midnight and peak&end after midnight
if (tp[i] < ts[i]) then begin
  tp[i] = (data[i].day + 1 - data[0].day) * 1440 + data[i].tp_hrs * 60 + data[i].tp_min
  te[i] = (data[i].day + 1 - data[0].day) * 1440 + data[i].te_hrs * 60 + data[i].te_min
endif

; for flares with start&peak b/f midnight and end after midnight
if (te[i] < ts[i]) then te[i] = (data[i].day + 1 - data[0].day) * 1440 + data[i].te_hrs * 60 + data[i].te_min
endif else begin
; then the month has changed
if (flag eq 0B) then begin
days_of_last_month = data[i-1].day - data[0].day + 1
flag = 1B
endif
ts[i] = (days_of_last_month + data[i].day - 1) * 1440 + data[i].ts_hrs * 60 + data[i].ts_min
tp[i] = (days_of_last_month + data[i].day - 1) * 1440 + data[i].tp_hrs * 60 + data[i].tp_min
te[i] = (days_of_last_month + data[i].day - 1) * 1440 + data[i].te_hrs * 60 + data[i].te_min

; for flares with start b/f midnight and peak&end after midnight
if (tp[i] < ts[i]) then begin
  tp[i] = (days_of_last_month + data[i].day + 1 - 1) * 1440 + data[i].tp_hrs * 60 + data[i].tp_min
  te[i] = (days_of_last_month + data[i].day + 1 - 1) * 1440 + data[i].te_hrs * 60 + data[i].te_min
endif

; for flares with start&peak b/f midnight and end after midnight
if (te[i] < ts[i]) then te[i] = (days_of_last_month + data[i].day + 1 - 1) * 1440 + data[i].te_hrs * 60 + data[i].te_min
endelse
endfor

; tr : array that contains the minutes of the rise phase of the events
; bg : array for background flux for each event as defined by Wheatland 2010 ( bg is average flux for duration=tr before ts )
; there are cases where bg > peak_flux -> for those don't subtract bg
tr = tp - ts
bg = fltarr(nrows)
real_peak = fltarr(nrows)
count = 0B
for i=0B,nrows-1 do begin
  bg[i] = mean(fluxs[ts[i]:ts[i]])
  if bg[i] lt peak_flux[i] then begin
    real_peak[i] = peak_flux[i] - bg[i]
    count++
  endif else begin
    real_peak[i] = peak_flux[i]
  endelse
endfor

; check for duplicates in the flares lmsal list and the all events list
count = 0
for i=0,nrows-2 do begin
  if (ts[i+1] lt te[i]) then begin
    print, i
    count++
  endif
endfor
print, 'duplicate flares in lmsal list= ',count

; SXR
; calculations for : start, end, peak times, peak flux, bg levels, bg-subtracted flux, bg-subtracted peak flux

\begin{verbatim}
start=ulonarr(nrows)
ende=ulonarr(nrows)
my_peak_time=ulonarr(nrows)
my_peak_flux=fltarr(nrows)
rise=ulonarr(nrows)
bg=fltarr(num_minutes)
bgsflare flux=fltarr(num_minutes)
bg2=fltarr(num_minutes)
flare2=fltarr(num_minutes)
sxr_flares=fltarr(num_minutes)
my_real_peak=fltarr(nrows)
c1=0
c2=0
for i=0,nrows-1 do begin
  ; find the start of flare -> 1st point before tp where fluxs is less that the 2 fluxs values preceding it
  ; the second condition with the equality of 3 adjacent values if added for periods when the long channel is at a constant
  ; level due to
  ; a) the background is so low that GOES is at 10^-8 for all points in a given time period
  ; b) instrumental anomalies cause GOES to falsely report a constant flux level for all points in a given period (step-wise
  ; increments)
  ; if the start you find is after the lmsal start you are in a 'double or multiple' flare -> go to lmsal start and look for a start
  before that
  pointer=tp[i]
  repeat pointer-- until ( (fluxs[pointer-1] gt fluxs[pointer] and fluxs[pointer-2] gt fluxs[pointer-1]) or (fluxs[pointer-1] eq
  fluxs[pointer] and fluxs[pointer-2] eq fluxs[pointer-1]) )
  start[i]=pointer
  if (start[i] gt ts[i]) then begin
    pointer=ts[i]
    repeat pointer-- until ( (fluxs[pointer-1] gt fluxs[pointer] and fluxs[pointer-2] gt fluxs[pointer-1]) or (fluxs[pointer-1]
    eq fluxs[pointer] and fluxs[pointer-2] eq fluxs[pointer-1]) )
  endif
  start[i]=pointer
  ; find the end of the flare -> 1st point after tp where fluxs is less that the 2 fluxs values following it
  ; if the end you find is before the lmsal end you are in a 'double or multiple' flare -> go to lmsal end and look for an end
  after that
  pointer=tp[i]
  repeat pointer++ until ( (fluxs[pointer+1] gt fluxs[pointer] and fluxs[pointer+2] gt fluxs[pointer+1]) or (fluxs[pointer+1]
  eq fluxs[pointer] and fluxs[pointer+2] eq fluxs[pointer+1]) )
  ende[i]=pointer
  if (ende[i] lt te[i]) then begin
    pointer=te[i]
    repeat pointer++ until ( (fluxs[pointer+1] gt fluxs[pointer] and fluxs[pointer+2] gt fluxs[pointer+1]) or
    (fluxs[pointer+1] eq fluxs[pointer] and fluxs[pointer+2] eq fluxs[pointer+1]) )
  endif
  ende[i]=pointer
  ; find max flux b/w your start,ende times -> this is the real peak (sometimes lmsal has this wrong)
  array=fltarr(ende[i]-start[i]+1)
  array=fluxs[start[i]:ende[i]]
  my_peak_time[i]=start[i]+where(array eq max(array))
  my_peak_flux[i]=max(array)
  ; wheatland's background subtraction method -> steady background equal to the mean of flux for as many points as rise
  time lasts
  rise[i]=my_peak_time[i]-start[i]
  bg[start[i]-5:ende[i]+5]=mean(fluxs[start[i]:rise[i]:start[i]])
  ; calculate the bg-subtracted flare flux
  ; if bg > fluxs -> assume the flare hasn't started yet and set new flux to 0.0
  for j=start[i],ende[i] do begin
    if (fluxs[j] gt bg[j]) then bgsub_flares_flux[j]=fluxs[j]-bg[j] else bgsub_flares_flux[j]=0.0
  endfor
  ; my background subtraction method -> linear interpolation of points between the flux values at start and ende of flare
  diff=fluxs[ende[i]-start[i]]
  dy=diff/(ende[i]-start[i])
  bg2[start[i]]=fluxs[start[i]]
\end{verbatim}
for j=start[i]+1,ende[i]-1 do bg2[j]=bg2[j-1]+dy
bg2[ende[i]]=fluxs[ende[i]]
; calculate the bg-subtracted flare flux
if bg > fluxs -> assume the flare hasn't started yet and set new flux to 0.0
for j=start[i],ende[i] do begin
  if (fluxs[j] gt bg2[j]) then flare2[j]=fluxs[j]-bg2[j] else flare2[j]=0.0
endfor
; for each flare choose b/w bgsub_flare_flux(wheatland method) and flare2(my method) according to which method
; produces the greatest peak
; graphical inspection of the flare # plots shows that in some cases wheatland's method is better and in some others my
; method is better
; the best method for each flare is the one producing greater flux values for the duration of the flare
if (flare2[my_peak_time[i]] gt bgsub_flare_flux[my_peak_time[i]]) then begin
  sxr_flare[start[i]:ende[i]]=flare2[start[i]:ende[i]]
  my_real_peak[i]=flare2[my_peak_time[i]]
c1++
endif else begin
  sxr_flare[start[i]:ende[i]]=bgsub_flare_flux[start[i]:ende[i]]
  my_real_peak[i]=bgsub_flare_flux[my_peak_time[i]]
c2++
endelse
endfor

;-----------------------------------------------------------------------------------------------
; plot the 1-min timeseries in the vicinity of each of the lmsal flares
; overplot all calculated quantities
for k=0,0 do begin
  create the x axis array with the appropriate times to plot each flare
  x = ulonarr(te[k]-ts[k]+120)
x[0] = ts[k]-60
for i=1,te[k]-ts[k]+119 do x[i]=x[i-1]+1
window, k, xsize=2000, ysize=1000
!p.color=cgcolor("black")
!p.background=cgcolor("white")
plot, x , fluxs[ts[k]-60:te[k]+60], psym=5, charsize=2
  ; /ylog, yrange=[min(flare2[start[k]+1:ende[k]-1]),max(fluxs[ts[k]-60:te[k]+60])]
!p.color=cgcolor("red")
oplot, [ts[k],tp[k],te[k]],[fluxs[ts[k]],fluxs[tp[k]],fluxs[te[k]]]
!p.color=cgcolor("green")
oplot, [start[k],my_peak_time[k],ende[k]], [fluxs[start[k]],my_peak_flux[k],fluxs[ende[k]]]
!p.color=cgcolor("blue")
oplot, x, bg[ts[k]-60:te[k]+60]
!p.color=cgcolor("purple")
oplot, x, bgsub_flare_flux[ts[k]-60:te[k]+60]
!p.color=cgcolor("green")
oplot, x, bg2[ts[k]-60:te[k]+60]
!p.color=cgcolor("orange")
oplot, x, flare2[ts[k]-60:te[k]+60]
stringk=str(k)
filename = filepath('flare #'+stringk+.png', root_dir="/home", subdir=['gkromyda','IDLWorkspace80','Default', ar,'bg sub plots-sxr'])
;write_png, filename, tvrd(/true)
endfor

;-----------------------------------------------------------------------------------------------
; create fluxh3min array containing the 3-min binned hxr timeseries
; 1-min binned hxr are fluctuating a lot so bin more to help you with hxr flare identification
how_many_left=num_minutes mod 3
if (how_many_left eq 0) then begin
  fluxh3min=fltarr(num_minutes/3)
j=0UL
for i=0UL,(num_minutes/3-1) do begin
fluxh3min[i]=mean(fluxh[j:j+2])
j=j+3
endif
endif
if (how_many_left eq 1) then begin
  fluxh3min=fltarr((num_minutes+2)/3)
j=0UL
  for i=0UL,(num_minutes+2)/3-2 do begin
    fluxh3min[i]=mean(fluxh[j:j+2])
    j=j+3
  endfor
endif
if (how_many_left eq 2) then begin
  fluxh3min=fltarr((num_minutes+1)/3)
j=0UL
  for i=0UL,(num_minutes+1)/3-3 do begin
    fluxh3min[i]=mean(fluxh[j:j+2])
    j=j+3
  endfor
endif
size=size(fluxh3min)
size_3min=size[1]

; HXR ; calculations for : start, end, peak times, peak flux, bg levels, bg-subtracted flux, bg-subtracted peak flux
starth=ulonarr(nrows)
endeh=ulonarr(nrows)
my_peak_timeh=ulonarr(nrows)
my_peak_fluxh=fltarr(nrows)
riseh=ulonarr(nrows)
bgh=fltarr(num_minutes)
bg2h=fltarr(num_minutes)
flare2h=fltarr(num_minutes)
lxhr_flare=fltarr(num_minutes)
my_real_peakh=fltarr(nrows)
c1=0
c2=0
for i=0,nrows-1 do begin
  peakhelp=start[i]+where(fluxh[start[i]:ende[i]] eq max(fluxh[start[i]:ende[i]]))
  pointer=peakhelp[0]/3
  repeat pointer-- until ( (fluxh3min[pointer-1] gt fluxh3min[pointer]) or (fluxh3min[pointer-1] eq fluxh3min[pointer]) )
  starthelp=pointer
  pointer=peakhelp[0]/3
  repeat pointer++ until ( (fluxh3min[pointer+1] gt fluxh3min[pointer]) or (fluxh3min[pointer+1] eq fluxh3min[pointer]) )
  endehelp=pointer
  st_pos=where(fluxh[3*starthelp-2:3*starthelp+2] eq min(fluxh[3*starthelp-2:3*starthelp+2]))
  starth[i]=3*starthelp-2+st_pos[0]
  end_pos=where(fluxh[3*endehelp-2:3*endehelp+2] eq min(fluxh[3*endehelp-2:3*endehelp+2]))
  endeh[i]=3*endehelp-2+end_pos[0]
  find max flux b/w your starth, endeh times
  array=fltarr(endeh[i]-starth[i]+1)
  array=fluxh[starth[i]:ende[i]]
  peak_pos=where(array eq max(array))
  my_peak_timeh[i]=starth[i]+peak_pos[0]
  my_peak_fluxh[i]=max(array)
; wheatland's background subtraction method -> steady background equal to the mean of flux for as many points as rise
time lasts
riseh[i]=my_peak_timeh[i]-starth[i]
bgh[starth[i]-2:ende[h]+2]=mean(fluxh[starth[i]-riseh[i]:starth[i]])
; calculate the bg-subtracted flare flux
; if bg > fluxs -> assume the flare hasn't started yet and set new flux to 0.0
for j=starth[i],ende[h] do begin
  if (fluxh[j] gt bg[h]) then bgsub_flare_fluxh[j]=fluxh[j]-bgh[j] else bgsub_flare_fluxh[j]=0.0
endfor
; my background subtraction method -> linear interpolation of points between the flux values at start and ends of flare
diff=fluxh[ende[h]]-fluxh[starth[i]]
dy=diff/(ende[h]-starth[i])
bg2h[starth[i]]=fluxh[starth[i]]
for j=starth[i]+1,ende[h]-1 do bg2h[j]=bg2h[j-1]+dy
bg2h[ende[h]]=fluxh[ende[h]]
; calculate the bg-subtracted flare flux
; if bg > fluxs -> assume the flare hasn't started yet and set new flux to 0.0
for j=starth[i],ende[h] do begin
  if (fluxh[j] gt bg2h[j]) then flare2h[j]=fluxh[j]-bg2h[j] else flare2h[j]=0.0
endfor
; calculate the bg-subtracted flare flux
; if bg > fluxh -> assume the flare hasn't started yet and set new flux to 0.0
for j=starth[i],ende[h] do begin
  if (fluxh[j] gt bg2h[j]) then flare2h[j]=fluxh[j]-bg2h[j] else flare2h[j]=0.0
endfor
; for each flare choose b/w bgsub_flare_flux(wheatland method) and flare2(my method) according to which method produces the greatest peak
; graphical inspection of the flare # plots shows that in some cases wheatland's method is better and in some others my method is better
; the best method for each flare is the one producing greater Flux values for the duration of the flare
if (flare2h[my_peak_timeh[i]] gt bgsub_flare_fluxh[my_peak_timeh[i]]) then begin
  hxr_flare[starth[i]:ende[h]]=flare2h[starth[i]:ende[h]]
  my_real_peakh[i]=flare2h[my_peak_timeh[i]]
c1++
endif else begin
  hxr_flare[starth[i]:ende[h]]=bgsub_flare_fluxh[starth[i]:ende[h]]
  my_real_peakh[i]=bgsub_flare_fluxh[my_peak_timeh[i]]
c2++
endelse
endfor

; plot the 1-min timeseries in the vicinity of each of the hxr flares as identified by the calculations part above
; overplot all calculated quantities
for k=0,0 do begin
; create the x axis array with the appropriate times to plot each flare
x = thonarr(ende[h]-starth[k]+120)
x[0] = starth[k]-60
for i=1,ende[h]-starth[k]+119 do x[i]=x[i-1]+1
window, k, xsize=2000, ysize=1000
!p.color=cgcolor("black")
!p.background=cgcolor("white")
plot, x , fluxh[starth[k]-60:ende[h]+60], psym=5, charsize=2
;/ylog, yrange=min(flare2h[starth[k]+1:ende[h]-1]),max(fluxh[starth[k]-60:ende[h]+60])
!p.color=cgcolor("red")
oplot, [starth[k],my_peak_timeh[k],ende[h]], [fluxh[starth[k]],my_peak_fluxh[k],fluxh[ende[h]]]
!p.color=cgcolor("blue")
oplot, x, bg[starth[k]-60:ende[h]+60]
!p.color=cgcolor("purple")
oplot, x, bgsub_flare_fluxh[starth[k]-60:ende[h]+60]
!p.color=cgcolor("green")
oplot, x, bg2h[starth[k]-60:ende[h]+60]
!p.color=cgcolor("orange")
oplot, x, flare2h[starth[k]-60:ende[h]+60]
; stringk=string(k)
; filename = filepath('flare #'+stringk+'.png', root_dir='/home', subdir=['gkromyda','IDLWorkspace80','Default', ar, bg sub plots-hxr'])
; write_png, filename, tvrd(/true)
endfor

;----------------------------------------------------------------------------------------
; AUTOCORRELATION ANALYSIS

timelags = indgen(1000)
result1 = A_CORRELATE(sxr_flare, timelags)
result2 = A_CORRELATE(hxr_flare, timelags)
; create a zeroline X and a 1/e line Y
X = lonarr(2)
Y = fltarr(2)
X[0] = 0
X[1] = 3000
Y[0] = 1.00/EXP(1)
Y[1] = 1.00/EXP(1)
; AUTOCORRELATION PLOT
!p.background=cgcolor("white")
!p.color=cgcolor("black")
window, 0, xsize=2000, ysize=1000
PLOT, timelags, result1, title='Autocorrelation Function for the Flare Timeseries of AR10848', xtitle='Time (min)', ytitle='Autocorr.', charsize=1.5
!p.color=cgcolor("blue")
oplot, timelags, result2
!p.color=cgcolor("red")
oplot, X, Y
!p.color=cgcolor("black")
xyouts, 800, 500, 'ACF of 1-8A timeseries in black', charsize=1.5, /device
!p.color=cgcolor("blue")
xyouts, 800, 450, 'ACF of 0.5-4A timeseries in blue', charsize=1.5, /device
filename = filepath('flare_acf_10848.png', root_dir='/home', subdir=['gkromyda','IDLWorkspace80','Default','ptyxiakh plots','acf plots'])
write_png, filename, tvrd(/true)
stop

;---------------------------------------------------------------------------------------------
; check for overlapping flares produced by my flare start,ende finding algorithm
for i=0,nrows-2 do if (start[i+1] lt ende[i]) then print, 'overlap in flares ',i,' -',i+1
;---------------------------------------------------------------------------------------------
; thermal energy
; E_T = \[ 3kT(t)EM(t) \]/n
; n=10^10 cm-3 because the t and em tables are created using chianti with this density (White et.al.2005)
E_T = 4.141974*10^9*(29.0)*t_em_calc.temp*t_em_calc.emis_meas
energy_start=ulonarr(nrows)
energy_end=ulonarr(nrows)
flare_ther_ener=dblarr(nrows)
flare_dur=ulonarr(nrows)
flare_ener_peak=dblarr(nrows)
; find the energy_start and energy_end times following the rule -> start looking at the E_T array from the start point of
; the sxr flare
; if you find a zero followed by a non-zero save the non-zero position as the energy_start
; continue until you find a non-zero followed by a zero value and save the non-zero as the energy_end
; the above rule is used because inspection of the E_T array shows that :
; a) the 1st non-zero E_T value for a flare is at the start time of the sxr flare or at the next 1-min value
; b) for adjacent flares with ende[i]=start[i+1] there is always a zero E_T value b/w the flares
; this is why we demand for 1 zero value to be found in order to call it an energy_end
; c) there are some flares that look as if they have stopped but then after many minutes produce 2-3 non-zero values
; we consider this as a spurious phenomenon and do not take these late points into consideration for the energy_end
for i=0,nrows-2 do begin
  ; find the flare duration -> number of minutes b/w start and ende where E_T is non-zero

count_zeros=0
for j=start[i],ende[i] do if (E_T[j] eq 0.0) then count_zeros++
flare_dur[i]=ende[i]-start[i]+1-count_zeros

; variation of the definition of flare_dur: the duration of the rise time of the hxr flare event (Lu & Hamilton 91 suggestion)
; flare_dur[i]=my_peak_timeh[i]-starth[i]+1

; activate to see how many zero valued minutes you have for each flare
; print, count_zeros
; find the energy_start -> time after start[i] of 1st non-zero E_T value
estart_pos=start[i]
repeat estart_pos++ until (E_T[estart_pos] ne 0.0)
energy_start[i]=estart_pos
; find the energy_end -> time before ende[i] of 1st non-zero E_T value
eend_pos=ende[i]
repeat eend_pos-- until (E_T[eend_pos] ne 0.0)
energy_end[i]=eend_pos

; flare_ener_peak[i]=max(E_T[energy_start[i]:energy_end[i]])
energy_sum=0.00
for j=energy_start[i],energy_end[i] do energy_sum=energy_sum+mean([E_T[j],E_T[j+1]])
flare_ther_ener[i]=60*energy_sum
endfor

;-------------------------------------------------------------------------------------------
; waiting time
; e.x. wait_time[10] corresponds to the time interval between the end of flare #10 and the start of flare #11
; my flare start,ende finding algorithm can create overlapping flares when te[i] is a couple of minutes before ts[i+1]
; for such consequent flares the 2 points greater that are required to identify the end might be inside the next flare
; or the 2 points greater that are required to identify the start might be inside the previous flare
; usually the overlap is for 2-min
; since for most cases the '2 points greater' condition helps with flare identification to correct false early starts, we
; prefer to keep the condition
; the problem with overlapping flares is only with the calculation of the waiting time b/w them
; the existence of zeros in E_T helps to divide overlapping flares when calculating the energy_start and energy_end times
; therefore for most overlapping flares the overlap is corrected automatically
; for those overlapping flares that are still not corrected we manually set them to have zero waiting time
wait_time=lonarr(nrows-1)
for i=0,nrows-2 do begin
  wait_time[i]=energy_start[i+1]-energy_end[i]
  if (wait_time[i] lt 0) then wait_time[i]=0
endfor

;-------------------------------------------------------------------------------------------
; BTW1988: the power spectrum S(f) is done for the instantaneous energy dissipation rate
; this is the S(f) for which they expect S(f)=f^(-2+a)=f^(-b), where D(t)=t^(-a)
; their simulations show for D=2 => b=1.57, for D=3 => b=1.08
C3 = fltarr(num_minutes)
C3 = FFT(E_T)
P3 = abs(C3)^(2.0)
v = fltarr(num_minutes/2+1)
v[0] = 0
for i=1,num_minutes/2 do v[i] = i/(60.0*num_minutes)

; FFT PLOT of P3 only with slopes fitted
!p.color=cgcolor("black")
window, 3, xsize=2000, ysize=1000
plot, v[1:num_minutes/2], P3[1:num_minutes/2], /xlog, /ylog, xminor=9, yminor=9, psym=10, charsize=1.5,
title='Spectral Power of Thermal Energy Release Rate Timeseries of '+ar, xtitle='Frequency v (Hz)', ytitle='Power P(v)'
result1 = regress(alog10(v[80:4500]), alog10(P3[80:4500]), sigma=sigma1, yfit=y1)
; prepare data for cumulative number dist. plots
; for each peak count how many peaks are greater
; do this both for measured data and background subtracted data
sorted_peak_flux = peak_flux[sort(peak_flux)]
sorted_real_peak_sxr = my_real_peak[sort(my_real_peak)]
sorted_real_peak_hxr = my_real_peakh[sort(my_real_peakh)]
sorted_ther_ener = flare_ther_ener[sort(flare_ther_ener)]
sorted_flare_dur = flare_dur[sort(flare_dur)]
sorted_flare_ener_peak = flare_ener_peak[sort(flare_ener_peak)]
sorted_wait_time = wait_time[sort(wait_time)]
number_greater_with_bg = lonarr(nrows)
number_greater_bgsub_sxr = lonarr(nrows)
number_greater_bgsub_hxr = lonarr(nrows)
number_greater_ther_ener = lonarr(nrows)
number_greater_flare_dur = lonarr(nrows)
number_greater_flare_peak = lonarr(nrows)
number_greater_wait_time = lonarr(nrows-1)
for i=0B,nrows-1 do begin
  sum1=0B
  sum2=0B
  sum3=0B
  sum4=0B
  sum5=0B
  sum6=0B
  sum7=0B
  for j=0,nrows-1 do begin
    if (peak_flux[j] gt sorted_peak_flux[i]) then sum1++
    if (my_real_peak[j] gt sorted_real_peak_sxr[i]) then sum2++
    if (my_real_peakh[j] gt sorted_real_peak_hxr[i]) then sum3++
    if (flare_ther_ener[j] gt sorted_ther_ener[i]) then sum4++
    if (flare_dur[j] gt sorted_flare_dur[i]) then sum5++
    if (flare_ener_peak[j] gt sorted_flare_ener_peak[i]) then sum6++
  endfor
  number_greater_with_bg[i]=sum1
  number_greater_bgsub_sxr[i]=sum2
  number_greater_bgsub_hxr[i]=sum3
  number_greater_ther_ener[i]=sum4
  number_greater_flare_dur[i]=sum5
  number_greater_flare_peak[i]=sum6
endfor
for i=0,nrows-2 do begin
  sum7=0B
  for j=0,nrows-2 do begin
    if (wait_time[j] gt sorted_wait_time[i]) then sum7++
    number_greater_wait_time[i]=sum7
  endfor
endfor

; RAW SXR & BGSUB SXR & BGSUB HXR DISTRIBUTION
window, 1 ,xsize=2000, ysize=1000
!p.background=cgcolor("white")
!p.color=cgcolor("black")
plot, sorted_peak_flux[0:nrows-2], number_greater_with_bg[0:nrows-2], /xlog, /ylog, xrange=[min(sorted_real_peak_hxr),max(sorted_peak_flux)], yrange=[10^(0.0),max(number_greater_with_bg)], psym=1, symsize=2, title='Cumulative Number Distribution for '+ar, xtitle='Background-Subtracted Peak Flux (W/m^2)', ytitle='Number greater', charsize=1.5
oplot, sorted_peak_flux[0:nrows-2], number_greater_with_bg[0:nrows-2]
!p.color=cgcolor("blue")
oplot, sorted_real_peak_sxr[0:nrows-2], number_greater_bgsub_sxr[0:nrows-2], psym=4, symsize=2
!p.color=cgcolor("green")
oplot, sorted_real_peak_hxr[0:nrows-2], number_greater_bgsub_hxr[0:nrows-2], psym=5, symsize=2
oplot, sorted_real_peak_hxr[0:nrows-2], number_greater_bgsub_hxr[0:nrows-2]
; if /xlog,/ylog are removed, try to fit a model of the form y=a_ox^(a_1)+a_2 using comfit
A = [1,-1,0]
result=comfit(sorted_real_peak_sxr[0:nrows-2], number_greater_bgsub_sxr[0:nrows-2], A, /geometric, itmax=100, yfit=y)
!p.color=cgcolor("red")
oplot, sorted_real_peak_sxr[0:nrows-1], y
; fit_sxr1 = regress(alog10(sorted_real_peak_sxr[10:70]), alog10(number_greater_bgsub_sxr[10:70]), yfit=ys1, sigma=ss1)
fit_sxr2 = regress(alog10(sorted_real_peak_sxr[70:nrows-5]), alog10(number_greater_bgsub_sxr[70:nrows-5]), yfit=ys2, sigma=ss2)
; fit_hxr1 = regress(alog10(sorted_real_peak_hxr[13:62]), alog10(number_greater_bgsub_hxr[13:62]), yfit=yh1, sigma=sh1)
fit_hxr2 = regress(alog10(sorted_real_peak_hxr[72:nrows-5]), alog10(number_greater_bgsub_hxr[72:nrows-5]), yfit=yh2, sigma=sh2)
; !p.color=cgcolor("red")
oplot, sorted_real_peak_sxr[10:70], 10^(ys1)
oplot, sorted_real_peak_sxr[70:nrows-5], 10^(ys2)
oplot, sorted_real_peak_hxr[13:62], 10^(yh1)
oplot, sorted_real_peak_hxr[72:nrows-5], 10^(yh2)
; slope_s1=string(fit_sxr1-1,format='(f0.2)')
sigma_s1=string(ss1,format='(f0.2)')
slope_s2=string(fit_sxr2-1,format='(f0.2)')
sigma_s2=string(ss2,format='(f0.2)')
slope_h1=string(fit_hxr1-1,format='(f0.2)')
sigma_h1=string(sh1,format='(f0.2)')
slope_h2=string(fit_hxr2-1,format='(f0.2)')
sigma_h2=string(sh2,format='(f0.2)')
; !p.color=cgcolor("black")
xyouts, 200, 200, 'long channel freq.dist.', charsize=2, /device
xyouts, 200, 190, '---------------', charsize=2, /device
xyouts, 200, 160, 'slope1='+slope_s1+'+/-'+sigma_s1, charsize=2, /device
xyouts, 200, 130, 'slope2='+slope_s2+'+/-'+sigma_s2, charsize=2, /device
; xyouts, 500, 200, 'short channel freq.dist.', charsize=2, /device
xyouts, 500, 190, '---------------', charsize=2, /device
xyouts, 500, 160, 'slope1='+slope_h1+'+/-'+sigma_h1, charsize=2, /device
xyouts, 500, 130, 'slope2='+slope_h2+'+/-'+sigma_h2, charsize=2, /device
filename = filepath('x-ray peaks_'+ar+'.png', root_dir='~/home', subdir=['x-ray peaks','IDLWorkspace80','Default','ptyxiakh plots','cum.dist.'])
write_png, filename, tvrd(/true)

;ENERGY DISTRIBUTION
window, 2, xsize=2000, ysize=1000
!p.color=cgcolor("black")
plot, sorted_ther_ener[1:nrows-2], number_greater_ther_ener[1:nrows-2], /xlog, /ylog, psym=5, symsize=2,
title='Cumulative Number Distribution for Thermal Energies of '+ar+' flares', xtitle='Thermal Energies (ergs)', ytitle='Number greater', charsize=1.5
oplot, sorted_ther_ener[1:nrows-2], number_greater_ther_ener[1:nrows-2]

fit_energy1= regress(alog10(sorted_ther_ener[8:65]), alog10(number_greater_ther_ener[8:65]), yfit=ye1, sigma=se1)
fit_energy2= regress(alog10(sorted_ther_ener[65:nrows-2]), alog10(number_greater_ther_ener[65:nrows-2]), yfit=ye2, sigma=se2)

!p.color=cgcolor("red")
oplot, sorted_ther_ener[8:65], 10^(ye1)
oplot, sorted_ther_ener[65:nrows-2], 10^(ye2)

slope_e1=string(fit_energy1-1,format='(f0.2)')
sigma_e1=string(se1,format='(f0.2)')
slope_e2=string(fit_energy2-1,format='(f0.2)')
sigma_e2=string(se2,format='(f0.2)')

!p.color=cgcolor("black")
xyouts, 230, 200, 'Freq.Dist. slopes', charsize=2, /device
xyouts, 200, 190, '--------------', charsize=2, /device
xyouts, 200, 160, 'slope1='+slope_e1+'+/-'+sigma_e1, charsize=2, /device
xyouts, 200, 130, 'slope2='+slope_e2+'+/-'+sigma_e2, charsize=2, /device

filename = filepath('thermal energy_'+ar+'.png', root_dir='/home', subdir=['gkromyda','IDLWorkspace80','Default','ptyxiakh plots','cum.dist.'])
write_png, filename, tvrd(/true)

; BTW 1988 : from the exponent beta of the fourier plot and the exponent tau of the energy distribution plot, the 
exponent gamma of growth rate can be calculated
; gamma=(2-beta-tau)/(tau-2)
gamma1 = (2 + result1 + fit_energy1 - 1)/(fit_energy1 + 1 - 2)
print, 'gamma1=',gamma1

gamma2 = (2 + result1 + fit_energy2 - 1)/(fit_energy2 + 1 - 2)
print, 'gamma2=',gamma2
stop

; DURATION DISTRIBUTION
window, 3, xsize=2000, ysize=1000
!p.color=cgcolor("black")
plot, sorted_flare_dur[0:nrows-2], number_greater_flare_dur[0:nrows-2], /xlog, /ylog, psym=5, symsize=2,
title='Cumulative Number Distribution for Duration of '+ar+' flares', xtitle='Duration (min)', ytitle='Number greater',
charsize=1.5
oplot, sorted_flare_dur[0:nrows-2], number_greater_flare_dur[0:nrows-2]

fit_dur2= regress(alog10(sorted_flare_dur[15:57]), alog10(number_greater_flare_dur[15:57]), yfit=yd2, sigma=sd2)
fit_dur3= regress(alog10(sorted_flare_dur[57:nrows-3]), alog10(number_greater_flare_dur[57:nrows-3]), yfit=yd3, sigma=sd3)

!p.color=cgcolor("red")
oplot, sorted_flare_dur[15:57], 10^(yd2)
oplot, sorted_flare_dur[57:nrows-3], 10^(yd3)

slope_d2=string(fit_dur2-1,format='(f0.2)')
sigma_d2=string(sd2,format='(f0.2)')
slope_d3=string(fit_dur3-1,format='(f0.2)')
sigma_d3=string(sd3,format='(f0.2)')

!p.color=cgcolor("black")
xyouts, 230, 200, 'Freq.Dist. slopes', charsize=2, /device
xyouts, 200, 190, '--------------', charsize=2, /device
xyouts, 200, 140, 'slope1='+slope_d2+'+/-'+sigma_d2, charsize=2, /device
xyouts, 200, 120, 'slope2='+slope_d3+'+/-'+sigma_d3, /device
filename = filepath('duration_200+_ar+.png', root_dir='~/home', subdir=['gkromyda','IDLWorkspace80','Default','ptyxiakh plots','cum.dist.'])
write_png, filename, tvrd(/true)

; ENERGY PEAKS DISTRIBUTION
window, 4, xsize=2000, ysize=1000
!p.color=cgcolor("black")
plot, sorted_flare_ener_peak[0:nrows-2], number_greater_flare_peak[0:nrows-2], /xlog, /ylog, psym=5, symsize=2,
title='Cumulative Number Distribution for Peaks in Thermal Energy of '+'ar+' flares', xtitle='Thermal Energy Rate (ergs/s)',
ytitle='Number greater', charsize=1.5
oplot, sorted_flare_ener_peak[0:nrows-2], number_greater_flare_peak[0:nrows-2]

fit_p1= regress(alog10(sorted_flare_ener_peak[8:72]), alog10(number_greater_flare_peak[8:72]), yfit=yp1, sigma=sp1)
fit_p2= regress(alog10(sorted_flare_ener_peak[72:nrows-2]), alog10(number_greater_flare_peak[72:nrows-2]),
yfit=yp2, sigma=sp2)

!p.color=cgcolor("red")
oplot, sorted_flare_ener_peak[8:72], 10^(yp1)
oplot, sorted_flare_ener_peak[72:nrows-2], 10^(yp2)

slope_p1=string(fit_p1-1,format='(f0.2)')
sigma_p1=string(sp1,format='(f0.2)')
slope_p2=string(fit_p2-1,format='(f0.2)')
sigma_p2=string(sp2,format='(f0.2)')

!p.color=cgcolor("black")
xyouts, 230, 200, 'Freq.Dist. slopes', charsize=2, /device
xyouts, 200, 190, '------------', charsize=2, /device
xyouts, 200, 160, 'slope1='+slope_p1+'+/-'+sigma_p1, charsize=2, /device
xyouts, 200, 140, 'slope2='+slope_p2+'+/-'+sigma_p2, charsize=2, /device
filename = filepath('energy peaks_200+_ar+.png', root_dir='~/home',
subdir=['gkromyda','IDLWorkspace80','Default','ptyxiakh plots','cum.dist.'])
write_png, filename, tvrd(/true)

; these are to create points on the waiting times plot that show what were the waiting times the flares of the 2nd slope of
the thermal energy plot
pos_2nd_slope_flares = where(flare_ther_ener ge sorted_ther_ener[65])
size = size(pos_2nd_slope_flares)
num_greater_2nd_slope_flares = lonarr(n)
for i=0,n-1 do begin
    wt_2nd_slope_flares[i] = wait_time[pos_2nd_slope_flares[i]-1]
pos_in_sorted = where(sorted_wait_time eq wt_2nd_slope_flares[i])
    num_greater_2nd_slope_flares[i] = number_greater_wait_time[pos_in_sorted]
endfor

; WAITING TIME DISTRIBUTION
window, 5, xsize=2000, ysize=1000
!p.color=cgcolor("black")
plot, sorted_wait_time[0:nrows-3], number_greater_wait_time[0:nrows-3], /xlog, /ylog, psym=5, symsize=2,
title='Cumulative Number Distribution for Waiting Time of '+'ar+' flares', xtitle='Waiting Time (min)',
ytitle='Number greater', charsize=1.5
!
p.color=cgcolor("green")
oplot, wait_2nd_slope_flares[num_greater_2nd_slope_flares, psym=6, symsize=2

fit_w1= regress(sorted_wait_time[0:47], alog10(number_greater_wait_time[0:47]), yfit=yw1, sigma=sw1)
fit_w2 = regress(sorted_wait_time[48:nrows-4], alog10(number_greater_wait_time[48:nrows-4]), yfit=yw2, sigma=sw2)

!p.color = cgcolor("red")
oplot, sorted_wait_time[0:47], 10**(yw1)
oplot, sorted_wait_time[48:nrows-4], 10**(yw2)

wt_w1 = string(-alog10(exp(1))/fit_w1, format='(f0.2)')
sigma_w1 = string((alog10(exp(1))*sw1)/(fit_w1^2.0), format='(f0.2)')
wt_w2 = string(-alog10(exp(1))/fit_w2, format='(f0.2)')
sigma_w2 = string((alog10(exp(1))*sw2)/(fit_w2^2.0), format='(f0.2)')

!p.color = cgcolor("black")
xyouts, 210, 200, 'average waiting times', charsize=2, /device
xyouts, 200, 190, '---------------', charsize=2, /device
xyouts, 200, 160, 'slope1='+wt_w1+'+/-'+sigma_w1, charsize=2, /device
xyouts, 200, 140, 'slope2='+wt_w2+'+/-'+sigma_w2, charsize=2, /device
filename = filepath('wt_2nd_slope'+ar+'.png', root_dir='/home', subdir=['gkromyda','IDLWorkspace80','Default','ptyxiakh plots','cum.dist.'])
write_png, filename, tvrd(/true)

stop

; WAIT TIME vs. ENERGY PLOT
window, 6, xsize=2000, ysize=1000
!p.background = cgcolor("white")
!p.color = cgcolor("black")
plot, flare_ther_en[1:nrows-1], wait_time, /xlog, psym=2, symsize=2, title='Waiting Time vs. Thermal Energy for '+ar, xtitle='Thermal Energy (ergs)', ytitle='Waiting time (min)', charsize=1.5
filename = filepath('w_E_'+ar+'.png', root_dir='/home', subdir=['gkromyda','IDLWorkspace80','Default','ptyxiakh plots','correlations'])
write_png, filename, tvrd(/true)

; ENERGY vs. ENERGY PEAKS PLOT
window, 7, xsize=2000, ysize=1000
!p.background = cgcolor("white")
!p.color = cgcolor("black")
plot, flare_ener_peak, flare_ther_ener, /xlog, /ylog, psym=2, symsize=2, title='Thermal Energy vs. Peak Thermal Energy Release Rate for '+ar, xtitle='Thermal Energy Release Rate (ergs/s)', ytitle='Thermal Energy (ergs)', charsize=1.5

!p.color = cgcolor("red")
oplot, flare_ener_peak, 10**(yep)

slope_ep = string(fit_E_p, format='(f0.2)')
sigma_ep = string(sep, format='(f0.2)')
!p.color = cgcolor("black")
xyouts, 900, 150, 'slope= '+slope_ep+'+/-'+sigma_ep, charsize=2, /device
filename = filepath('E_P_'+ar+'.png', root_dir='/home', subdir=['gkromyda','IDLWorkspace80','Default','ptyxiakh plots','correlations'])
write_png, filename, tvrd(/true)

; ENERGY vs. DURATION PLOT
window, 8, xsize=2000, ysize=1000
!p.background = cgcolor("white")
!p.color = cgcolor("black")
plot, flare_dur, flare_ther_ener, /xlog, /ylog, psym=2, symsize=2, title='Thermal Energy vs. Duration for '+ar,
xtitle='Duration of Thermal Energy Release (min)', ytitle='Thermal Energy (ergs)', charsize=1.5
fit_E_d=regress(alog10(flare_dur),alog10(flare_ther_ener),yfit=yed,sigma=sed)
!p.color=cgcolor("red")
oplots, flare_dur, 10^**(yed)
;
slope_ed=string(fit_E_d,format='(f0.2)')
sigma_ed=string(sed,format='(f0.2)')
!p.color=cgcolor("black")
xyouts, 900, 150, 'slope= '+slope_ed+'+/-'+sigma_ed, charsize=2, /device
filename = filepath('E_d_'+ar+'.png', root_dir='/home', subdir=['gkromyda','IDLWorkspace80','Default','ptyxiakh plots','correlations'])
write_png, filename, tvrd(/true)

;-----------------------------------------------------------------------------------------------

; ENERGY PEAKS vs. DURATION PLOT
window, 9 ,xsize=2000, ysize=1000
!p.background=cgcolor("white")
!p.color=cgcolor("black")
plot, flare_dur, flare_ener_peak, /xlog, /ylog, psym=2, symsize=2, title='Peak Thermal Energy Release Rate vs. Duration for '+ar, xtitle='Duration of Thermal Energy Release (min)', ytitle='Peak Thermal Energy Release Rate (ergs/s)', charsize=1.5
fit_p_d=regress(alog10(flare_dur),alog10(flare_ener_peak),yfit=ypd,sigma=spd)
!p.color=cgcolor("red")
oplots, flare_dur, 10^**(ypd)
;
slope_pd=string(fit_p_d,format='(f0.2)')
sigma_pd=string(spd,format='(f0.2)')
!p.color=cgcolor("black")
xyouts, 900, 150, 'slope= '+slope_pd+'+/-'+sigma_pd, charsize=2, /device
filename = filepath('P_d_'+ar+'.png', root_dir='/home', subdir=['gkromyda','IDLWorkspace80','Default','ptyxiakh plots','correlations'])
write_png, filename, tvrd(/true)

;-----------------------------------------------------------------------------------------------

; FOURIER PLOTS
;
; FFT coefficients stored in C1 and C2 arrays
; Power Spectral Density stored in P1 and P2 arrays where P(v)=|X(v)|^2, X(v)=Re{X(v)} + iIm{X(v)}
; A POINT TO REMEMBER : you tried with the fluxs and fluxh arrays too and the slopes where in similar ranges as
; with sxr_flare and hxr_flare but quite increased
; ex a -2.61 became a -2.98 and a -3.27 became a -3.34
; also the fft plots with sxr_flare and hxr_flare look 'clearer' that's why I use them
C1 = fltarr(num_minutes)
C1 = FFT(sxr_flare)
C2 = fltarr(num_minutes)
C2 = FFT(hxr_flare)
P1 = abs(C1)^(2.0)
P2 = abs(C2)^(2.0)

; v array contains the real frequencies that correspond to the FFT coefficients
; valid calculation for either even or odd num_minutes b/c
; num_minutes:even -> last element of v array is (num_minutes/2)/(60.0*num_minutes)=1/(2*60.0) as expected for the
; fft of an even array
; num_minutes:odd -> num_minutes/2 computes to (num_minutes-1)/2 b/c division is integer/integer -> last element of
; v array is (num_minutes-1)/2*60.0*num_minutes
; as expected for the fft of an odd array
v = fltarr(num_minutes/2+1)
v[0] = 0
for i=1,num_minutes/2 do v[i] = i/(60.0*num_minutes)
FFT PLOT of P1 and P2 together
with x-axis in real frequencies
!p.background=cgcolor("white")
!p.color=cgcolor("black")
window, 0, xsize=2000, ysize=1000
plot, v[1:num_minutes/2], P1[1:num_minutes/2], /xlog, /ylog, xminor=9, yminor=9, psym=10, charsize=1.5,
title='Power Spectrum of the X-ray Flare Timeseries of '+'ar, xtitle='Frequency v (Hz)', ytitle='Power P(v)'
!p.color=cgcolor("blue")
oplot, v[1:num_minutes/2], P2[1:num_minutes/2]
!p.color=cgcolor("black")
xyouts, 200, 150, '1-8A flare timeseries in black', charsize=1.5, /device
!p.color=cgcolor("blue")
xyouts, 200, 125, '0.5-4A flare timeseries in blue', charsize=1.5, /device
filename = filepath('Both channels_''ar+.png', root_dir='/home', subdir=['gkromyda','IDLWorkspace80','Default','ptyxiakh plots','fourier plots'])
write_png, filename, tvrd(/true)

FFT PLOT of P1 only with slopes fitted
!p.color=cgcolor("black")
window, 1, xsize=2000, ysize=1000
plot, v[1:num_minutes/2], P1[1:num_minutes/2], /xlog, /ylog, xminor=9, yminor=9, psym=10, charsize=1.5,
title='Spectral Power of the 1-8A X-ray Flare Timeseries of '+'ar, xtitle='Frequency v (Hz)', ytitle='Power P(v)'
result1 = regress(alog10(v[1:330]), alog10(P1[1:330]), sigma=sigma1, yfit=y1)
result2 = regress(alog10(v[330:num_minutes/2]), alog10(P1[330:num_minutes/2]), sigma=sigma2, yfit=y2)
!p.color=cgcolor("red")
oplot, v[1:330], 10^(y1)
oplot, v[330:num_minutes/2], 10^(y2)
slope_1=string(result1, format='(f0.2)')
sigma_1=string(sigma1, format='(f0.2)')
slope_2=string(result2, format='(f0.2)')
sigma_2=string(sigma2, format='(f0.2)')
!p.color=cgcolor("black")
xyouts, 200, 200, 'Slope in Frequency range [7*10^(-7),2.4*10^(-4)]= '+'slope_1+''/+-''+sigma_1, charsize=1.5, /device
xyouts, 200, 175, 'Slope in Frequency range [2.4*10^(-4),8.3*10^(-3)]= '+'slope_2+''/+-''+sigma_2, charsize=1.5, /device
filename = filepath('sxr_flare_''ar+.png', root_dir='/home', subdir=['gkromyda','IDLWorkspace80','Default','ptyxiakh plots','fourier plots'])
write_png, filename, tvrd(/true)

FFT PLOT of P2 only with slopes fitted
!p.color=cgcolor("black")
window, 2, xsize=2000, ysize=1000
plot, v[1:num_minutes/2], P2[1:num_minutes/2], /xlog, /ylog, xminor=9, yminor=9, psym=10, charsize=1.5,
title='Spectral Power of the 0.5-4A X-ray Flare Timeseries of '+'ar, xtitle='Frequency v (Hz)', ytitle='Power P(v)'
!p.color=cgcolor("blue")
oplot, v[1:num_minutes/2], P2[1:num_minutes/2], psym=10
result1 = regress(alog10(v[1:553]), alog10(P2[1:553]), sigma=sigma1, yfit=y1)
result2 = regress(alog10(v[553:num_minutes/2]), alog10(P2[553:num_minutes/2]), sigma=sigma2, yfit=y2)
!p.color=cgcolor("red")
oplot, v[1:553], 10^(y1)
oplot, v[553:num_minutes/2], 10^(y2)
slope_1=string(result1, format='(f0.2)')
sigma_1=string(sigma1, format='(f0.2)')
slope_2=string(result2, format='(f0.2)')
sigma_2=string(sigma2, format='(f0.2)')
!p.color=cgcolor("black")
xyouts, 200, 200, 'Slope in Frequency range [7*10^(-7),4*10^(-4)]= '+'slope_1+''/+-''+sigma_1, charsize=1.5, /device
xyouts, 200, 175, 'Slope in Frequency range [4*10^(-4),8.3*10^(-3)]= '+'slope_2+''/+-''+sigma_2, charsize=1.5, /device
filename = filepath('hxr_flare_''ar+.png', root_dir='/home', subdir=['gkromyda','IDLWorkspace80','Default','ptyxiakh plots','fourier plots'])
write_png, filename, tvrd(/true)
; the part below can be used both for ar_10848_t_em.emis_meas/ar_10848_t_em.temp plots and E_T plots
; n : size of array
size = size(ar_10848_t_em.emis_meas)
 n = size[1]
; activate this only for ar_10848_t_em, so than it can be plotted on a ylog axis
; I choose 10^(-8.0) b/c all non-zero values of em are greater than 10^(-8.0)
for i=0,n-1 do if (ar_10848_t_em.emis_meas[i] eq 0D) then ar_10848_t_em.emis_meas[i]=10^(-8.0)
for i=0,0 do begin
  ;...;
  
  number=string(i,format='(i02)')
  !p.background=cgcolor("white")
  !p.color=cgcolor("black")
  plot, x, ar_10848_t_em.emis_meas[energy_start[i]-20:energy_end[i]+20], title='Emission Measure Profile of flare#'+number+' of '+ar, xtitle='Time (min)', ytitle='EM (cm^-3)', psym=2, symsize=2
  oplot, x, ar_10848_t_em.emis_meas[energy_start[i]-20:energy_end[i]+20]
  ; activate the red and green points only for the E_T plots
  !p.color=cgcolor("red")
  oplot, [energy_start[i],energy_end[i]], [ar_10848_t_em.emis_meas[energy_start[i]],ar_10848_t_em.emis_meas[energy_end[i]]], psym=2, symsize=4
  !p.color=cgcolor("green")
  oplot, [start[i],ende[i]], [ar_10848_t_em.emis_meas[start[i]],ar_10848_t_em.emis_meas[ende[i]]], psym=2, symsize=4
  filename = filepath('em_flare#'+number+'.png', root_dir='/home', subdir=['gkromyda','IDLWorkspace80','Default','AR10848','em_plots'])
  write_png, filename, tvrd(/true)
endfor

stop

dend
Appendix B

X-ray Peaks Distributions

The distributions are presented in pairs:

- First plot: The distribution of events from the region’s flare list
- Second plot: The distribution of events from the region’s all events list
Appendix B. X-ray Peaks Distributions

Cumulative Number Distribution for AR10848

- Long channel freq. dist.: slope1 = \(-1.96^{+/-}0.02\), slope2 = \(-1.83^{+/-}0.03\)
- Short channel freq. dist.: slope1 = \(-1.59^{+/-}0.02\), slope2 = \(-1.41^{+/-}0.04\)
Appendix B. X-ray Peaks Distributions

Cumulative Number Distribution for AR10960

- long channel freq dist:
  - slope1 = -1.54+-0.01
  - slope2 = -2.56+-0.16

- short channel freq dist:
  - slope1 = -1.43+-0.00
  - slope2 = -2.34+-0.26

Cumulative Number Distribution for AR10963

- long channel freq dist:
  - slope1 = -1.82+-0.01
  - slope2 = -3.36+-0.18

- short channel freq dist:
  - slope1 = -1.69+-0.01
  - slope2 = -4.61+-1.02
Appendix B. X-ray Peaks Distributions

Cumulative Number Distribution for AR10978

Cumulative Number Distribution for AR11029

Long channel freq dist: slope1 = -1.45+/−0.01, slope2 = -2.18+/−0.03
Short channel freq dist: slope1 = -1.46+/−0.01, slope2 = -1.91+/−0.04

Long channel freq dist: slope1 = -1.07+/−0.02, slope2 = -4.04+/−0.40
Short channel freq dist: slope1 = -1.57+/−0.01, slope2 = -4.43+/−0.24
Appendix B. X-ray Peaks Distributions

Cumulative Number Distribution for AR11158

Cumulative Number Distribution for AR11271
Appendix B. X-ray Peaks Distributions

Cumulative Number Distribution for AR12192

- Long channel freq dist: slope $=-1.74\pm0.01$
- Short channel freq dist: slope $=-1.62\pm0.00$
Appendix C

Flare Thermal Energy Distributions

The distributions are presented in pairs:

- First plot: The distribution of events from the region’s flare list
- Second plot: The distribution of events from the region’s all events list
Appendix C. Flare Thermal Energy Distributions

Cumulative Number Distribution for Thermal Energies of AR10848 Flares

- Freq Dist, slopes
- slope 1 = -1.55 +/- 0.01
- slope 2 = -2.27 +/- 0.07

Cumulative Number Distribution for Thermal Energies of AR10846, AR10847, AR10848, AR10849 Flares

- Freq Dist, slopes
- slope 1 = -1.66 +/- 0.01
- slope 2 = -2.39 +/- 0.07
Appendix C. Flare Thermal Energy Distributions

Cumulative Number Distribution for Thermal Energies of AR1029 flares

- \text{Freq. Dist. slopes} -

slope1 = -1.49 +/− 0.01
slope2 = -2.46 +/− 0.07

Cumulative Number Distribution for Thermal Energies of AR11029 and flares from non-registered active regions

- \text{Freq. Dist. slopes} -

slope1 = -1.52 +/− 0.01
slope2 = -2.41 +/− 0.07
Appendix C. Flare Thermal Energy Distributions

Cumulative Number Distribution for Thermal Energies of AR1158 flares

slope = -1.85 \pm 0.01

Cumulative Number Distribution for Thermal Energies of AR1157, AR1158, AR1159, AR1161, AR1162 flares

slope = -1.70 \pm 0.01
Appendix C. Flare Thermal Energy Distributions

Cumulative Number Distribution for Thermal Energies of AR12192 Flares

Cumulative Number Distribution for Thermal Energies of AR12191–AR12195, AR12197, AR12201 Flares

slopes = -1.41 ± 0.00

slopes = -1.43 ± 0.00
Appendix C. Flare Thermal Energy Distributions

Cumulative Number Distribution for Thermal Energies of AR12529 Flares

slope = 1.74 ± 0.01

Cumulative Number Distribution for Thermal Energies of AR12526, AR12528, AR12532 Flares

slope = 1.78 ± 0.01
Appendix D

Thermal Energy Peaks Distributions

The distributions are presented in pairs:

- First plot: The distribution of events from the region’s flare list
- Second plot: The distribution of events from the region’s all events list
Appendix D. Thermal Energy Peaks Distributions

Cumulative Number Distribution for Peaks in Thermal Energy of AR10848 flares

- FreqDist. slopes -
  slope1 = -1.39 +/- 0.02
  slope2 = -3.45 +/- 0.29
Cumulative Number Distribution for Peaks in Thermal Energy of AR10830 flares

slope = -1.48 +/- 0.00

Cumulative Number Distribution for Peaks in Thermal Energy of AR10863 flares

slope1 = -1.50 +/- 0.01
slope2 = -2.99 +/- 0.16
Appendix D. Thermal Energy Peaks Distributions

Cumulative Number Distribution for Peaks in Thermal Energy of AR10578 flares

Freq Dist: $slope = -1.82 \pm 0.01$

Cumulative Number Distribution for Peaks in Thermal Energy of AR11023 flares

Freq Dist: $slope1 = -1.78 \pm 0.02$
Freq Dist: $slope2 = -0.82 \pm 0.04$
Appendix D. Thermal Energy Peaks Distributions

Cumulative Number Distribution for Peaks in Thermal Energy of AR11158 Flares

slope = -1.77 +/- 0.02

Cumulative Number Distribution for Peaks in Thermal Energy of AR12192 Flares

slope = -1.56 +/- 0.01
Appendix E

Duration Distributions

The distributions are presented in pairs:

- First plot: The distribution of events from the region’s flare list
- Second plot: The distribution of events from the region’s all events list
Appendix E. Duration Distributions

Cumulative Number Distribution for Duration of AR10848 flares

- Freq Dist. slopes -

slope1 = -2.22 ± 0.02
slope2 = -3.59 ± 0.10
Cumulative Number Distribution for Duration of AR10930 flares

slope1 = -1.55 +/- 0.01
slope2 = -2.38 +/- 0.02

Cumulative Number Distribution for Duration of AR10960 flares

slope1 = -2.22 +/- 0.02
slope2 = -7.81 +/- 0.46
Appendix E. Duration Distributions

Cumulative Number Distribution for Duration of AR10963 Flares

- Frequent slopes -
  - slope1 = -1.61 +/- 0.02
  - slope2 = -2.76 +/- 0.04
  - slope3 = -7.00 +/- 0.58

Cumulative Number Distribution for Duration of AR10978 Flares

- Frequent slopes -
  - slope1 = -1.97 +/- 0.03
  - slope2 = -3.96 +/- 0.13
Appendix E. Duration Distributions

Cumulative Number Distribution for Duration of AR11271 flares

Cumulative Number Distribution for Duration of AR12182 flares

- Freq Dist. slopes -

slope1 = -1.68 +/- 0.02
slope2 = -2.70 +/- 0.05

slope1 = -1.63 +/- 0.01
slope2 = -2.39 +/- 0.02
slope3 = -3.88 +/- 0.23
Appendix E. Duration Distributions

Cumulative Number Distribution for Duration of AR12519 flares

**Freq Dist. slopes**
- slope 1: $-2.25 \pm 0.02$
- slope 2: $-3.54 \pm 0.29$
Appendix F

Waiting Time Distributions
Appendix F. Waiting Time Distributions

Cumulative Number Distribution for Waiting Time of AR10848 Flares

Average waiting times:

slope = 106.82 ± 1.71
slope = 312.02 ± 6.72

Cumulative Number Distribution for Waiting Time of AR10930 Flares

Average waiting times:

slope = 102.03 ± 0.85
slope = 258.16 ± 23.15
Appendix F. Waiting Time Distributions

![Graph of Cumulative Number Distribution for Waiting Time of AR10863 flares]

Average waiting times:
- slope 1 = 26.63 ± 1.06
- slope 2 = 64.49 ± 1.13

![Graph of Cumulative Number Distribution for Waiting Time of AR10878 flares]

Average waiting times:
- slope 1 = 126.28 ± 1.75
- slope 2 = 685.76 ± 5.93
Appendix F. Waiting Time Distributions

Cumulative Number Distribution for Waiting Time of AR11029 Flares

- Average waiting time
- slope1 = 83.11 +/- 1.92
- slope2 = 280.93 +/- 16.39

Cumulative Number Distribution for Waiting Time of AR1158 Flares

- Average waiting time
- slope = 133.36 +/- 1.25
Appendix F. Waiting Time Distributions

Cumulative Number Distribution for Waiting Time of AR11271 Flares

Average waiting times
slope1 = 191.34 +/- 2.68
slope2 = 554.97 +/- 20.88

Cumulative Number Distribution for Waiting Time of AR12539 Flares

Average waiting times
slope1 = 266.04 +/- 20.37
slope2 = 287.26 +/- 5.15
Appendix G

Fourier Plots
Appendix G. Fourier Plots

Spectral Power of Thermal Energy Release Rate Timeseries of AR10848

Slope in Frequency range \([5.0\times10^{-5},3.5\times10^{-3}]\) = \(-1.04\pm0.02\)

Spectral Power of the 1–8 Å X-ray Background Timeseries of AR10848

Slope in Frequency range \([2.4\times10^{-4},0.5\times10^{-2}]\) = \(-2.17\pm0.01\)
Appendix G. Fourier Plots

Spectral Power of the 1-8A X-ray Flare Timeseries of AR10848

Slope in frequency range [$7\times10^{-7}$, $2.4\times10^{-6}$] = $-0.23\pm0.06$
Slope in frequency range [$2.4\times10^{-6}$, $6.3\times10^{-5}$] = $-2.0\pm0.1$

Spectral Power of the 0.5-4A X-ray Flare Timeseries of AR10848

Slope in frequency range [$7\times10^{-7}$, $4.1\times10^{-6}$] = $-0.23\pm0.05$
Slope in frequency range [$4.1\times10^{-6}$, $8.3\times10^{-5}$] = $-3.27\pm0.02$
Appendix G. Fourier Plots

Spectral Power of Thermal Energy Release Rate Timeseries of AR10330

Slope in frequency range \([1.4\times10^{-4},6.5\times10^{-3}]\) = \(-2.54\pm0.02\)

Spectral Power of the 1–8A X-ray Background Timeseries of AR10330

Slope in frequency range \([4.5\times10^{-5},0.5\times10^{-3}]\) = \(-1.66\pm0.00\)
Appendix G. Fourier Plots

Spectral Power of Thermal Energy Release Rate Timeseries of AR10360

Slope in frequency range $[5.4 \times 10^{-5}, 5.5 \times 10^{-3}]$ is $-1.47 \pm 0.01$

Spectral Power of the 1-8A X-ray Background Timeseries of AR10360

Slope in frequency range $[2.1 \times 10^{-4}, 5.0 \times 10^{-3}]$ is $-2.00 \pm 0.02$
Appendix G. Fourier Plots

Spectral Power of Thermal Energy Release Rate Timeseries of AR10963

Slope in frequency range $[0.6 \times 10^{-5}, 0.5 \times 10^{-3}]$ is $-1.54 \pm 0.01$

Spectral Power of the 1-8A X-ray Background Timeseries of AR10963

Slope in frequency range $[1.5 \times 10^{-4}, 0.5 \times 10^{-3}]$ is $-1.96 \pm 0.02$
Appendix G. Fourier Plots

Spectral Power of the 1–84 X-ray Flare Timelines of AR10963

Slope in Frequency range \([7\times10^{-7},4\times10^{-4}]\) = -0.38 +/- 0.06
Slope in Frequency range \([4\times10^{-4},1.9\times10^{-3}]\) = -0.07 +/- 0.07
Slope in Frequency range \([1.9\times10^{-3},6.5\times10^{-3}]\) = -3.8 +/- 0.04

Spectral Power of the 0.5–44 X-ray Flare Timelines of AR10963

Slope in Frequency range \([7\times10^{-7},4\times10^{-4}]\) = -0.24 +/- 0.04
Slope in Frequency range \([4\times10^{-4},2.2\times10^{-3}]\) = -1.75 +/- 0.06
Slope in Frequency range \([2.2\times10^{-3},6.5\times10^{-3}]\) = -3.84 +/- 0.04
Appendix G. Fourier Plots

Spectral Power of Thermal Energy Release Rate Timeseries of AR10378

Slope in Frequency range $[3.9 \times 10^{-5}, 6.1 \times 10^{-3}]$ is $-1.31 \pm 0.01$

Spectral Power of the 1-5A X-ray Background Timeseries of AR10378

Slope in Frequency range $[5.9 \times 10^{-4}, 8.5 \times 10^{-3}]$ is $-2.09 \pm 0.01$
Appendix G. Fourier Plots

Spectral Power of the 1–8A X-ray Flare Timeseries of AR10978

Slope in Frequency range [1x10^-7, 3x10^-4] = -0.14 +/- 0.07
Slope in Frequency range [-3x10^-4, 1x10^-3] = -1.95 +/- 0.06
Slope in Frequency range [1.2x10^-5, 6.5x10^-3] = -3.58 +/- 0.02

Spectral Power of the 0.5–44 X-ray Flare Timeseries of AR10978

Slope in Frequency range [7x10^-7, 3x10^-4] = -0.03 +/- 0.06
Slope in Frequency range [3x10^-4, 1.4x10^-3] = -1.07 +/- 0.07
Slope in Frequency range [1.1x10^-5, 6.5x10^-3] = -3.53 +/- 0.02
Appendix G. Fourier Plots

Spectral Power of Thermal Energy Release Rate Timeseries of AR11029

Slope in Frequency range $[2.5 \times 10^{-5} - 5 \times 10^{-3}]$ Hz = $-1.93 \pm 0.02$

Spectral Power of the 1-8A X-ray Background Timeseries of AR11029

Slope in Frequency range $[1.5 \times 10^{-4} - 8.5 \times 10^{-3}]$ Hz = $-2.21 \pm 0.02$
Appendix G. Fourier Plots

Spectral Power of Thermal Energy Release Rate Timeseries of AR1158

Slope in Frequency range $[3.0 \times 10^{-4}, 0.5 \times 10^{-3}]$ = $-2.07 \pm 0.02$

Spectral Power of the 1–8A X-ray Background Timeseries of AR1158

Slope in Frequency range $[1.1 \times 10^{-4}, 0.5 \times 10^{-3}]$ = $-1.89 \pm 0.01$
Appendix G. Fourier Plots

Spectral Power of the 1–8A X-ray Flare Timeseries of AR11271

Spectral Power of the 0.5–44 X-ray Flare Timeseries of AR11271
Appendix G. Fourier Plots

Spectral Power of Thermal Energy Release Rate Timeseries of AR12192

Slope in frequency range $[3.0 \times 10^{-5}, 3.6 \times 10^{-4}]$: $-1.86 \pm 0.10$

Spectral Power of the 1–5A X-ray Background Timeseries of AR12192

Slope in frequency range $[7.7 \times 10^{-5}, 0.5 \times 10^{-4}]$: $-1.91 \pm 0.01$
Appendix G. Fourier Plots

Spectral Power of Thermal Energy Release Rate Timeseries of AR12529

Slope in Frequency range [1.0×10⁻⁵,1.5×10⁻³] Hz = -1.13±0.03

Spectral Power of the 1-8A X-ray Background Timeseries of AR12529

Slope in Frequency range [5.1×10⁻⁵,0.5×10⁻³] Hz = -2.12±0.01
Bibliography


