First-passage time of Markov processes to moving barriers

Figure 1. The random process hits the moving barrier $Y(t)$, the time of first appearance in the $(x, y)$-plane. The trajectory is a domain enclosing the initial point $(x, y)$ and its boundary is $T$. The time of first passage being marked is $T$.

The theory of first-exit times for multidimensional diffusion processes (Dynkin (1965); Gihman and Skorohod (1972)) thus enables us to obtain differential equations for the moments $y$. With $Y(t)$ we may now consider the vector random process $Y(t)$ which satisfies the degenerate system of first-order stochastic equations and treat the first-passage-time problem for $X$ as a first-exit-time problem for the vector $(X, Y)$ in the plane. The separate trajectories of $X$ and $Y$ are sketched in Figure 1, appear as in Figure 2 when plotted in the $Y$-plane. We therefore see that the time of first passage of $X$ to $Y$ is the time of first exit of $(X, Y)$ from all or some part of the half-plane, $y$. The theory of first-exit times for multidimensional diffusion processes thus enables us to obtain differential equations for the moments $y$.