Neuropercollation and Related Models of Criticalities

Péter Érdi\textsuperscript{1,2} Róbert Kozma\textsuperscript{3} Marko Puljic\textsuperscript{4} Judit Szente\textsuperscript{1}

\textsuperscript{1}Center for Complex Systems Studies, Kalamazoo College, Kalamazoo, Michigan, USA

\textsuperscript{2}Wigner Research Centre for Physics, Hung. Acad. Sci. Budapest, Hungary

\textsuperscript{3}University of Memphis, Memphis, TN, USA

\textsuperscript{4}Tulane University, New Orleans, LA, USA

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Black Swans and Dragon Kings
(N.N. Taleb vs. D. Sornette)

Types of rare events

- Extreme events are large in size and at the tail of the PDF.
- Black swans — observation from the same population distribution (power law)
- Dragon Kings — observation not related to the distribution of the majority of observations (too large and too many).
Black Swans and Dragon Kings

Identification

- Pisarenko & Sornette: Robust Statistical Tests of Dragon-Kings beyond Power Law Distributions
- Goal: to identify the existence of even a single anomalous extreme events in the tail of the distribution.
- We test the null hypothesis:

\[ H_0 : \text{all observations of the sample are generated by the same exponential distribution} \]

Using Pareto distribution instead of Exponential is also a valid approach.
Black Swans and Dragon Kings

DK test

- Consider the spacing of a random sample: \( y_k = x_{k:n} - x_{1:n} \)
  where \( x_i \) is the \( i^{th} \) order statistic of the random sample \( x \).
- \( y_k \) is also exponential with expected value \( \frac{1}{a \cdot k} \).
- \( z_k = k \cdot y_k \) is also exponentially distributed.
- We can build a test statistic based on the sum of \( z \) which is F-distributed.

\[
T = \frac{\sum_{i=1}^{r} z_i}{\sum_{j=i}^{n-r} z_{r+j}} \frac{n-r}{n-r}
\]

- p-values—dependent of the exponential parameter (or Pareto parameter)—can be calculated according to the F-test.
Black Swans and Dragon Kings

U test

- Using MLE and rule of thumb we obtain a sub-sample that is an incomplete *beta* distribution.
- We obtain p-values for each extreme observation.
- If there exist some ”small” values among the p-values, than there are some DKs at a 1-p confidence level.
Black Swans and Dragon Kings

Phase transitions as critical events

Event-size distribution and phase transitions in the neuropercolation model show similarities.

**Figure:** M. Puljic, R. Kozma
2005.

**Figure:** D. Sornette
Black Swans and Dragon Kings

Neural avalanches

Fig. 3. Avalanche size distribution for 100 configurations of scale free networks with $N = 4000$ neurons and different percentage $p_{\text{inhi}}$ of inhibitory synapses under strong random drive.

L. de Arcangelis
disinhibition induced DKs
Black Swans and Dragon Kings

Generation

- Self-organized criticality (P. Bak) $\rightarrow$ inherently unpredictable
- Intermittent criticality (D. Sornette) $\rightarrow$ there are precursors

Search for Generative Mechanisms:
sand-pile model (local) versus probabilistic long-range connections
Consider \( d \)-dimensional discrete tori. There are two possible states: active and passive. At a given time \( t \), \( x \) becomes active with probability \( p \). \( p \) is a function of the state of the closed neighborhood nodes \( \Gamma_x \). Governing rules vary in terms of connection types (range, probabilistic, directional), arousal functions, etc.
Neuropercolation

- Infinite homogeneous models show power law relationship or SOC behaviour: mean field and local majority models.
- Mixed models with local and long-range connections lead to intermittent criticality.
- Intermittent criticality produces extreme observations can be described as Dragon Kings.
- An occurring Dragon King in such case mean phase transition between two SOC models.
- Phase transitions and related critical parameters have been studied in each one.
Neuropercolation

Model specification

R. Kozma, M. Puljic

Fig. 3. Illustration of majority voting rule in 2-dimensional lattices with local neighborhood; full circles show 3 inactive nodes, open circles show 2 active nodes. Nodes $x$, $a$, $c$, and $d$ are excitatory, while node $b$ is inhibitory. In the given scenario, $s(x, t)$ is likely to remain inactive with probability $\omega$. 

\[
\begin{align*}
ax^+ & : (a \to x) \ 0 \to 0 \\
bx^- & : (b \to x) \ 1 \to 0 \\
cx^+ & : (c \to x) \ 0 \to 0 \\
dx^+ & : (d \to x) \ 1 \to 1 \\
xx^+ & : (x \to x) \ 0 \to 0 \\
\end{align*}
\]

4 out of 5 inactive

$s(x, t)$ is inactive w.p. $\omega$
Neuropercolation
Model specification
R. Kozma, M. Puljic

Randomly rewired edges
Phase Transitions

(S. Havlin)

- thermodynamic p.t.
- magnetic p.t.
- critical exponent, order parameter, correlation length, mean cluster size
Phase Transitions

Figure: $P_\infty$: probability that a site belongs to infinite cluster.

$P_\infty \propto (p - p_c)^\beta$
Phase Transitions

in 2D Majority Percolation (P. Balister, B. Bollobas, M. Walters)

On a two-dimensional torus of size $NN$ with $p$-majority transition rules one only needs two thin intersecting bands of active sites to ensure a high probability of reaching the high-density state in a short time.
Simulation Results

a: ferromagnetic regime ("order")
b: critical state
c: paramagnetic regime ("disorder")
Simulation Results

Figure: self-organized criticality
Simulation Results

Figure: intermittent criticality. Jumps from the mostly inactive system to mostly active, and vice versa, take a relatively short time.
Conclusions

• *neuronal avalanche* - a cascade of bursts of activity in neuronal networks - whose size distribution can be approximated by a power law, can be found, probably only under pathological conditions.

• however: as opposed to other aspects of Nature, the brain is a very particular structure: it has an abundancy of phase transitions, basically with theta frequency while in nature, they are rare.

• models of neuropercolations can be useful for understanding other systems (solar flares).
Reading I

P. Érdi
*Complexity Explained.*

D. Sornette and G. Ouillon
*Dragon-kings: mechanisms, statistical methods and empirical evidence,*
European Physical Journal, Special Topics 205, 1-26 (2012)

R Kozma
*Neuropercolation*
http://www.scholarpedia.org/article/Neuropercolation
R Kozma and M Puljic