Dynamic Threshold Modeling of Budget Changes

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About the Collaboration

László Zalányi
Lessons from Political Science

- the budget is a scalar number annually (the sum of many elements, i.e. categories)
- each year a new point is generated -> budget changes
- traditional pure incremental model of budget changes leads to Gaussian distribution.
- comparative statistics: budget change distribution deviates from the Gaussian (double-
  Pareto? generalized extreme distribution?)
- SKEW distributions? probably due to the existence of institutional friction.
US real outlays, percentage change

Jones et al, 2007: A General Empirical Law of Public Budgets: A Comparative Analysis. Figure 2a.
Lessons from Political Science

US total outlays, histogram

Jones et al, 2007: A General Empirical Law of Public Budgets: A Comparative Analysis. Figure 3a

- leptokurtic
- asymmetric
- bounded
- peak close to zero
Dynamic Modeling of Public Policy Decision Making Processes

We are building a macroscopic and microscopic models to explain how institutional mechanisms transform Gaussian to power law distributions by assuming:

- limited attention of political institutions
- friction by collective decision rules
- resistance to change
- error accumulation
- prioritized preferences, reprioritize
Dynamic Modeling of Public Policy Decision Making Processes

- Deterministic versus stochastic process
- Incremental model versus "avalanches"
- Is the initial distribution really Gaussian?
- Why and how to get SKEW distributions?
- Probably due to the existence of institutional friction.
A dynamic stimulus-response threshold model:

- $R_t$: response variable
- $S_t$: input signal, (Gaussian random variable)
- $C$: threshold
- $\lambda$: efficiency parameter or friction
- $\beta$: amplification parameter

$$R_t = \begin{cases} 
\beta S_t & \text{if } \sum_{t'=1}^{t-1} S_{t'} > C, \\
\lambda \beta S_t & \text{otherwise}
\end{cases}$$
A simulation result:

Figure 1: A leptokurtic distribution as a simulated response, comparing to a Gaussian distribution.
Multiple Threshold Models

- $C_1 = 3$; upper threshold
- $C_2 = 1.2$; middle threshold
- $C_3 = -4$; lower threshold
- $\lambda_{1p} = 0.03$; efficiency for the region of positive accumulated sum
- $\lambda_{1n} = 0.02$; efficiency for the region of negative accumulated sum
- $\lambda_{2p} = 0.3$; efficiency between the two positive threshold
- $\beta = 1$;
- $T$ denotes the time of last resetting to 0 of accumulated signal.
Multiple Threshold Models

\[ R_t = \begin{cases} 
\beta S_t & \text{if } \sum_{t'=T}^{t-1} S_{t'} > C_1, T = t \\
\lambda_{2p} \beta S_t & \text{if } C_1 > \sum_{t'=T}^{t-1} S_{t'} > C_2 \\
\lambda_{1n} \beta S_t & \text{if } \sum_{t'=T}^{t-1} S_{t'} < 0 \land S_t < 0 \\
\lambda_{1n} \beta S_t & \text{if } \sum_{t'=T}^{t-1} S_{t'} < C_3, T = t \\
\lambda_{1p} \beta S_t & \text{if } C_2 > \sum_{t'=T}^{t-1} S_{t'} > 0 \lor 0 > \sum_{t'=T}^{t-1} S_{t'} > C_3 \land S_t > 0 
\end{cases} \]
Figure 2: Transformation of the Gaussian input signals to skew distribution fitted to the French budget data. The upper figure shows the tenfold enlarged input Gaussian sample distribution and the budget distribution histogram. The lower figure shows the measured data histogram and the fitted model output histogram (solid line). Parameters of the fit: $C_1 = -3; C_2 = 1.6; C_3 = 4.6; \lambda_{1n} = 0.07; \lambda_{1p} = 0.04; \lambda_{2p} = 0.3.$
Figure 3: Transformation of the Gaussian input signals to skew distribution fitted to the US budget data. The upper figure shows the tenfold enlarged input Gaussian sample distribution and the budget distribution histogram. The lower figure shows the measured data histogram and the fitted model output histogram (solid line). Parameters of the fit: \( C_1 = -12; \ C_2 = 0; \ C_3 = 3; \ \lambda_{1n} = 0.01; \ \lambda_{1p} = 0.04; \ \lambda_{2p} = 0.3. \)
Conclusions and Some Open Problems

• Government budgets seem to be skew distributions

• Multiple threshold models should be better justified

• Can we design institutions that are more efficient? Do we want to?

• Can we get direct measurements of decision making costs?

• Why do budgets produce power laws but virtually all other distributions (such as GDP) are less extreme?