Knowledge Representation: Logic
Knowledge Representation

- formal logic
- rules
- concepts
- analogies
- images
- connections
Logic: some historical remarks

Aristotle’s logic

The Organon: theory of the syllogism

A deduction is speech (logos) in which, certain things having been supposed, something different from those supposed results of necessity because of their being so.
Syllogism:

Each of the "things supposed" is a premise (protasis) of the argument, and what "results of necessity" is the conclusion (sumperasma).

(categorical syllogism)

and much more ...

Aristotle’s logic was expressed in ordinary language so was still subject to the ambiguities of natural languages.
Logic: some historical remarks

Boole’s logic

George Boole: Laws of Thought (1854)

which formed a general symbolic method of logical inference. Given any propositions involving any number of terms, Boole showed how, by the purely symbolic treatment of the premises, to draw any conclusion logically contained in those premises.

Propositional logic: in which it is assumed that all propositions have a definite truth value, a proposition is either true or it is false.
Logic: some historical remarks

Gottlob Frege (1848-1925)

Founded the modern discipline of logic

axiomatization of the propositional logic

invention of "predicate logic" or "predicate calculus"

A predicate calculus is a formal system (a formal language and a method of proof) in which one can represent valid inferences among predications, i.e., among statements in which properties are predicated of objects. (see somewhat more later)
\[ A - n < b \]

\[ \frac{\gamma}{\beta} (0 + \Gamma = \delta_{\beta}) \]

\[ d < B \]

\[ n > 0 \]

\[ A \geq b \]

\[ \frac{\gamma}{\beta} (0 + \Gamma = \delta_{\beta}) \]

\[ d < B \]
Frege’s notation for first-order logic (1879)
Logic: some historical remarks

Logicism

logicism: mathematics is reducible to logic

Bertrand Russel

Kurt Gödel’s incompleteness theorems (1931)

showed that Frege’s logicist program was impossible
Formal logic

There is no unique definition:

http://www.rbjones.com/rbjpub/philos/logic/009.htm

One possible definition based on the concept of truth:

Logic is the study of necessary truths and of systematic methods for clearly expressing and rigourously demonstrating such truths.
Why?

A key feature of logic is that it is concerned with sound, a priori arguments those in which conclusions follow of necessity. The proposed definition seeks to make direct the connection between logic and necessity.
Another definition on methodological basis:

Logic can be defined as concerned with methods for reasoning. Logical systems are then formalisations of the proper methods and logical truths are those demonstrable by correct methods.
Formal logic

An **inference rule** is a method of deriving conclusions from premises.

When inference rules for a formal language are codified it becomes a formal logic.
Logic: Principles of Reasoning

Deductive: e.g. modus ponens

If p then q.
p
So q.

Inductive: uncertainty, e.g.

All KCollegs students are under 7 feet tall.
Modus tollens

If p then q.
q is false.
So p is false.

Modus tollens became somewhat legendary when it was used by Karl Popper in his proposed response to the problem of induction,
**Popper and the Problem of Induction**

Science does not actually rely on induction, developing the notion of falsification instead.

Popper replaced induction with deduction, in effect making modus tollens the centerpiece of his theory.

On this account, when assessing a theory one should pay greater heed to data which is in disagreement with the theory than to data which is in agreement with it.

Popper went further and stated that a hypothesis which does not allow of such experimental text is outside the bounds of science.
Predicate logic

Quantifiers: "All" "Some"

The most important knowledge representation language is arguably predicate logic (or strictly, first order predicate logic - there are lots of other logics out there to distinguish between). Predicate logic allows us to represent fairly complex facts about the world, and to derive new facts in a way that guarantees that, if the initial facts were true then so are the conclusions. It is a well understood formal language, with well-defined syntax, semantics and rules of inference.
Russel’s anecdote

A schoolboy, a parson and a mathematician are crossing from England into Scotland in a train. The schoolboy looks out and sees a black sheep and says, ”Oh! Look! Sheep in Scotland are black!” The parson, who is learned, says, ”No. Strictly speaking, all we can say is there is one sheep in Scotland that is black.”

The mathematician says, No, still not correct. All we can really say is that we know that in Scotland there exists at least one sheep, at least one side of which is black.”
Deductive reasoning

Deductive reasoning is the process of reaching a conclusion that is guaranteed to follow, if the evidence provided is true and the reasoning used to reach the conclusion is correct. The conclusion also must be based only on the evidence previously provided; it cannot contain new information about the subject matter. Deductive reasoning was first described by the ancient Greek philosophers such as Aristotle.
Lance J. Rips argues that certain inference principles are so central to our notion of intelligence and rationality that they deserve serious psychological investigation to determine their role in individuals’ beliefs and conjectures. Asserting that cognitive scientists should consider deductive reasoning as a basis for thinking, Rips develops a theory of natural reasoning abilities and shows how it predicts mental successes and failures in a range of cognitive tasks.
Advantages of logic

• Long history, well understood formalism

• Clear principles, rigorous

• Much representational power

• Quite good Computational power: A famous program: PROLOG
Disadvantages of logic

- Natural language is much more flexible than formal logic: not easy to formalize
- Restricted to verbal information
- Much reasoning is nonmonotonic: you can’t just add more beliefs deductively, but must subtract as well
- Potentially computationally explosive
- Most interesting kinds of reasoning are non-deductive
From Logic to Rules and Mental Models

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