Robust Image Representation and Decomposition by Laplacian Regularized Latent Low-Rank Representation

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Abstract—This paper discusses the image representation and decomposition problem using enhanced low-rank representation. Technically, we propose a Regularized Low-Rank Representation framework referred to as rLRR that is motivated by the fact that Latent Low-Rank Representation (LatLRR) delivers robust and promising results for image representation and feature extraction through recovering the hidden effects, but the locality among both similar principal and salient features to be encoded are not preserved in the original LatLRR formulation. To address this problem for obtaining enhanced performance, rLRR is proposed through incorporating an appropriate Laplacian regularization term that allows us to keep the local geometry of close features. Similar to LatLRR, rLRR decomposes a given data matrix from two directions by calculating a pair of low-rank matrices. But the similarities among principal features and salient features can be clearly preserved by rLRR. Thus the correlated features can be well grouped and the robustness of representations can also be effectively improved. The effectiveness of rLRR is examined by representation and recognition of real images. Results verified the validity of our presented rLRR technique.

Keywords—Low-rank representation; Regularization; Robust matrix decomposition; Feature extraction

I. INTRODUCTION

In numerous practical applications in the machine learning and computer vision communities, such as face recognition [3][16][17], system identification [25] and video surveillance [3][4], low-rank structures can be observed if high-dimensional data lie near a low-dimensional subspace [3], which arouses much attention and interests on the study of low-rank data recovery.

Principal Component Analysis (PCA) [5], and the recent recovery techniques [1][2][3][4][29][30] are essentially based on assuming that the data of interest are approximately drawn from the low-rank subspaces. Similar to sparse representation (SR) [12][16][17][24][28], LRR aims to seek the lowest-rank representation among all candidates that represent the row space of vectors [1]. But unlike SR, LRR finds the lowest-rank representation of all data jointly to well capture the global structures of data. LRR gets promising results for data recovery and correction, and is robust to noise, especially when the data sampling is sufficient and errors are properly bounded [1][2].

It is worth noting that the LRR formulation performs matrix recovery and decomposition by directly applying the observed data matrix as the dictionary, but the insufficient data and/or grossly corrupted observations may put its validity in jeopardy. To improve the performance of LRR, an enhanced version of LRR, referred to as Latent LRR (LatLRR)[2][10], was recently proposed for performing the hidden recovery. LatLRR aims at constructing the dictionary by employing both observed and unobserved hidden data. As a result, LatLRR is capable of resolving the insufficient data sampling problem supposing that the given dictionary is always sufficient to represent the subspaces. It is also elaborated that the hidden effects of the unobserved data can also be recovered through minimizing a nuclear norm problem approximately [2][3]. Different from LRR, LatLRR aims to reconstruct the given data matrix along two directions, i.e., column and row. Based on such strategy, reconstructing using both row and column information of data simultaneously can contribute together when facing missing values, implying that LatLRR is more robust to errors.

Similar to SR criterion, the lowest-rank representation aims at encoding originally compact points to similar reconstruction coefficients. But it should be noted that both LRR and LatLRR cannot guarantee such property due to the fact that they do not consider preserving such dependence relationships among the features in the iterative optimizations. As a result, the compact features may be encoded as dissimilar and even different low-rank coefficients, even if the dictionary is good. As observed from the representations of LRR and LatLRR for two similar faces in Figure 3, the reconstruction coefficients of the two similar faces are encoded as different ones. But note that such deficiency may potentially weaken the robustness of the low-rank representation for matrix recovery and decomposition in reality. In this work, we mainly address this issue to enhance the performance and robustness of LatLRR by endowing it the capability to preserve the similarity among compact features.
in the reconstruction process.

We highlight the major contributions of the work as follows. Firstly, a Regularized Low-Rank Representation, called rLRR, framework formulated on LatLRR is technically proposed. In this setting, a data-dependent Laplacian regularization term is added to the original LatLRR formulation for preserving the similarities among close features in the reconstruction process. With such similarity effectively preserved, compact features are able to be well grouped in vicinity of a dense region and the similarity among them can be preserved, as illustrated in Figures 2 and 3. That is, more discriminant and representative reconstruction coefficients are achieved using rLRR. It should be noted that the idea of regularization has been widely used in machine learning studies, e.g., [20][23][26]. Second, similar to the LatLRR formulation, rLRR also learns two-directional reconstruction coefficients to decompose the images and the effects of both observed and hidden data can be approximately recovered by convex nuclear minimization. Moreover, the computational issues of rLRR are also detailed.

The paper is summarized as follows. Section II first briefly reviews the LatLRR formulation. We in Section III propose rLRR mathematically. The simulation settings and results for evaluating the proposed technique on benchmark problems are described in Section IV. Finally, the paper is concluded.

II. LATENT LOW-RANK REPRESENTATION (LATLRR)

Given an observed data matrix \( X_o \in \mathbb{R}^{m \times n} \), where each column denotes an observation vector, LatLRR [10] aims to construct the dictionary using observed and unobserved hidden data. To enhance LRR, LatLRR optimizes the following problem:

\[
\min_{Z} \|Z\|_*, \quad \text{Subj. } X_o = [X_o, X_h] Z, \tag{1}
\]

where \( \|Z\|_* \) denotes the nuclear norm [15] of \( Z \), namely the sum of singular values, the concatenation (along column) of \( X_o \) and \( X_h \) is applied as the dictionary, and \( X_o \) denotes the unobserved hidden data issue. The above problem can resolve the insufficient data sampling issue, supposing \( [X_o, X_h] \) is always sufficient to represent the subspaces. Let \( Z_{oh} \) be the optimal solution to Eq.1 and \( Z_{oh} = [Z_{o,h}, Z_{h,o}] \) denote its row-wise partition such that \( Z_{o,h} \) and \( Z_{h,o} \) correspond to \( X_o \) and \( X_h \), respectively, then \( Z_{o,h} \) is a nontrivial block-diagonal matrix that can exactly reveal the true subspace membership even if the sampling of \( X_o \) is insufficient [10]. Then LatLRR recovers the affinity matrix \( Z_{oh} \) by \( X_o \).

Let \( U \Sigma V^T \) denote the skinny Singular Value Decomposition (SVD) of \( [X_o, X_h] \), and partition \( V = [V_o, V_h] \) such that \( X_o = U \Sigma V_o^T \) and \( X_h = U \Sigma V_h^T \), we have [2][10]:

\[
Z_{oh} = V_o^T V_o^T, \quad Z_{h,o} = V_h^T V_h^T, \tag{2}
\]

where notation \( ^T \) denotes the transpose of a matrix. Based on the formulations of Eq.2, one can further obtain

\[
X_o = [X_o, X_h] Z_{oh} = X_o Z_{o,h} + X_h Z_{h,o} = X_o Z_{o,h} + X_h V_h^T V_h^T = X_o Z_{o,h} + U \Sigma V_o^T V_o^T = X_o Z_{o,h} + L_{h,o} X_o \tag{3}
\]

where \( L_{h,o} = U \Sigma V_o^T V_o^T \Sigma^T \), and \( \Sigma^T \) is the inverse of \( \Sigma \). Then the hidden effects can be described as

\[
X_o = X_h Z_{o,h} + L_{h,o} X_o. \tag{4}
\]

Provided that both \( X_o \) and \( X_h \) are sampled from the same collection of low-rank subspaces whose union has rank \( r \); then it can be concluded that \( \text{rank}(Z_{o,h}) \leq r \) and \( \text{rank}(L_{h,o}) \leq r \). So, \( Z_{o,h} \) can be recovered with

\[
\min_{Z_{oh}} \text{rank}(Z_{o,h}) + \text{rank}(L_{h,o}), \quad \text{Subj. } X_o = X_h Z_{o,h} + L_{h,o} X_o. \tag{5}
\]

Note that solving Eq.5 is NP-hard. As suggested by [2][3], by simplifying \( Z_{o,h} \), \( L_{h,o} \) and \( X_o \) with \( Z \in \mathbb{R}^{m \times n} \), \( L \in \mathbb{R}^{n \times n} \) and \( X \in \mathbb{R}^{m \times n} \) respectively, Eq.5 can be relaxed as

\[
\min_{X, Z} \|X\|_F + \lambda \|Z\|_F, \quad \text{Subj. } X = XZ + LX + E \tag{6}
\]

when corrupted data is included, where \( \ell_1 \)-norm is chosen to characterize the sparse error term \( E \), and \( \lambda \) denotes a positive weighting parameter. It should be noted that the minimizer \( Z^* = [z_1^*, z_2^*, ..., z_n^*] \) (with respect to variable \( Z \)) is the recovery to \( Z_{o,h} \), thus \( Z^* \) can be considered as an “enhanced” lowest-rank representation and LatLRR may be more robust than the original LRR. Because the objective function of LatLRR is convex, for computational efficiency, the inexact Augmented Lagrange Multiplier (ALM) [7][29] is used to iteratively solve the problem. After the optimal solution \( (Z^*, E, E^*) \) is obtained, LatLRR algorithm decomposes \( X \) into a low-rank part \( XZ^* \), a low-rank part \( LX \), and a sparse part \( E^* \) that fits noise, where \( XZ^* \) and \( LX \) encode principal features and salient features of data respectively. As described in [2][10], matrix \( X^* \) can be applied to assign the edge weights over an undirected graph for measuring the similarity between vertices.

III. LAPLACIAN REGULARIZED LATENT LOW-RANK REPRESENTATION FRAMEWORK

A. Motivation

Let \( G = (V, E) \) be a graph consisting of \( N \) vertices \( \{v_i\}_{i=1}^N \), where \( v_i \) corresponds to \( X_i \) and \( E \) includes the edges \( e(i,j) \) that describe the similarity between \( v_i \) and \( v_j \). In local learning, heavy weights \( w_{ij} \), will be imposed on \( e(i,j) \) if \( v_i \) and \( v_j \) are “close”; otherwise small weights are incurred. Note that, for low-rank representation, if \( X_i \) and \( X_j \) are originally “close”, their encoded principal features, namely \( X_i \) and \( X_j \), shall also be compact as well. But such mutual dependence among principal features is not considered by LatLRR. In other words, the locality or similarity among features to be encoded is lost in LatLRR. So for real datasets with complex distributions, the principal features of “close” samples may be encoded to be far away in the feature representation subspace. Let \( \text{Tr} (\cdot) \) be trace operator. Because \( \{A_{ij}\} = \text{Tr} (A_{ij} A) \), let \( W \) endowed with entries \( W_{ij} \) denote a similarity matrix over \( G \), we can minimize

\[
\tilde{h}(Z) = \frac{1}{2} \sum_{ij} \| z - z \|_2^2 w_{ij} = \frac{1}{2} \sum_{ij} \text{Tr} ( (z - z)^T (z - z)) w_{ij} = \text{Tr} ( \sum_{ij} z^T z w_{ij} ) = \text{Tr} ( Z (W) Z ) \tag{7}
\]
to preserve the locality and similarity of the principal features, where $\|\cdot\|$ is $l_1$-norm, $D$ denotes the diagonal matrix whose entries are $D_{ii} = \sum_j W_{ij}$ and $D^{(0)} = D - W$ is a Laplacian matrix over the graph. It should be noted that a similar idea has been applied to regularize the LRR approach [18]. Analogously, the salient features, i.e., $L_x$ and $L_y$, of similar instances $x$ and $y$ should be also close. But again the locality and similarity among those salient features is also addressed by LatLRR in iterative optimizations. Because $\dot{A} = Tr(\dot{A}^T) = Tr(A^T)$, to preserve the local geometry or similarity of the salient features, the following criterion can be minimized:

$$
\dot{h}(L) = \sum_{i,j} \gamma_{ij} \|L_x - L_y\|_1 W_{ij} - \frac{1}{2} \sum_{i,j} \gamma_{ij} \|\{L_x - L_y\}(L_x - L_y)^T\| W_{ij}.
$$

(8)

In this work, we mainly focus on preserving the locality and similarity among both principal features and salient features in the reconstruction process at the same time, in addition to keeping the desirable low-rank characteristics of LatLRR.

B. The Objective Function

For a given data matrix $X = [X_1, X_2, \ldots, X_N] \in \mathbb{R}^{n \times N}$, rLRR seeks two low-rank matrices $Z = [Z_1, Z_2, \ldots, Z_N] \in \mathbb{R}^{n \times N}$ and $L = [L_1, L_2, \ldots, L_N] \in \mathbb{R}^{n \times N}$ to decompose $X$ in two directions by using bases in $Z$ and $L$. By including a Laplacian regularization $\|L\|_1$ to LatLRR for similarity preservation and imposing a nonnegative constraint on $Z$, rLRR solves the lowest-rank problems from

$$
\begin{align*}
\min_{X,L} \|X\|_F^2 + \|L\|_1 + \frac{\gamma}{2} \|Z\|_1, \quad \text{Subj } X = XL + E, \quad Z \geq 0, \quad (9)
\end{align*}
$$

where $\|X\|_F^2 = \sum_{i,j} (X_{ij} - \bar{X}_{ij})^2$ denotes the $l_2$ -norm [13] of $X$, because we expect to model the sample-specific corruptions and outliers. To preserve the locality and similarity among the principal and salient features, a natural choice for $\|L\|_1$ is

$$
\|L\|_1 = \dot{h}(Z) + \dot{h}(L) = \frac{1}{2} \sum_{i,j} \gamma_{ij} \|L_x - L_y\|_1 W_{ij} + \frac{1}{2} \sum_{i,j} \gamma_{ij} \|L_x - L_y\|_1 W_{ij}. \quad (10)
$$

Note that preserving the locality and similarity of principal features and salient features are equivalently important, so no parameter is added in Eq.10 to balance the tradeoff between $\dot{h}(Z)$ and $\dot{h}(L)$. By using the matrix interpretation according to Eqs.7 and 8, the problem of Eq.10 becomes

$$
\begin{align*}
\min_{X,L,E} \|X\|_F^2 + \frac{\gamma}{2} \|Z\|_1 + \frac{\gamma}{2} \|\{Z^T L^{(0)} Z\} + \|L X^{(0)} X^T L^T\|\|_2, \quad \text{Subj } X = XL + E, \quad Z \geq 0, \quad (11)
\end{align*}
$$

where terms $\|X\|_F^2$, $\|L\|_1$, and $\|\cdot\|_2$ are convex. It is also noted that $L^{(0)}$ is a graph Laplacian matrix, implying that the term $Z^T X^{(0)} X^T L^T$ is also convex. That is, the above problem in Eq.11 is convex. To facilitate the optimization, we first convert Eq.11 to the following equivalent one:

$$
\begin{align*}
\min_{X,L,E} \|X\|_F^2 + \frac{\gamma}{2} \|Z\|_1 + \frac{\gamma}{2} \|\{Z^T L^{(0)} Z\} + \|L X^{(0)} X^T L^T\|\|_2, \quad \text{Subj } X = XL + E, \quad Z \geq 0, \quad (12)
\end{align*}
$$

It is clear that the above problem is reduced to LatLRR when $\gamma = 0$ and $l_1$-norm is imposed on the sparse error matrix $E$. Thus the above problem can be similarly solved as LatLRR. In this work, the inexact ALM [7][29] is used to solve rLRR due to its effectiveness and efficiency. The convergence properties of the presented rLRR framework are similar as LatLRR [2][10][29]. The followings will detail the issues of computing the low-rank and sparse matrices.

C. Optimization for Low-rank Matrices

For the problem in Eq.12, the augmented Lagrangian function $\mathcal{A}(Z,S,L,E,Y,Z,J,\mu) = \|Z\|_2 + \gamma \|Z\|_1 + \gamma \|\{Z^T L^{(0)} Z\} + \|L X^{(0)} X^T L^T\|\|_2, \quad (13)$

where $Y_i \in \mathbb{R}^{n \times N}$, $Z_j \in \mathbb{R}^{n \times N}$, $Y_i \in \mathbb{R}^{n \times N}$ are Lagrange multipliers, and $\mu$ is a parameter. We first show how to optimize $J$. Note that

$$
\begin{align*}
\|Z\|_2^2 + \gamma \|Z\|_1 + \gamma \|\{Z^T L^{(0)} Z\} + \|L X^{(0)} X^T L^T\|\|_2, \quad (14)
\end{align*}
$$

Since $(1/2\mu)\|\cdot\|_2$ is a constant. Let $\tilde{\Phi}_i^Z = Z_i + (1/\mu_i)Y_i$ and $U, \Sigma, V^T$ denote the SVD of $\tilde{\Phi}_i^Z$, $J$ can be solved with the singular value thresholding (SVD) algorithm [6][8][9] as

$$
\begin{align*}
J_{i+1} = U \Omega_{[\mu_i]} \Sigma Y_i V^T. \quad (15)
\end{align*}
$$

Note that the optimization of $S_{i+1}$ is similar in spirit as optimizing $J_{i+1}$. Specifically, when solving the iterative $S_{i+1}$, $\|Y_i\|_1 + \gamma \|Z\|_1 + \gamma \|\{Z^T L^{(0)} Z\} + \|L X^{(0)} X^T L^T\|\|_2, \quad (16)$

where $\|X\|_2^2$ is a constant. Let $\tilde{\Phi}_i^S = S_i + (1/\mu_i)Y_i$ and $U, \Sigma, V^T$ be the SVD of $\tilde{\Phi}_i^S$, this optimization can also be solved by SVD [6][8][9] as

$$
\begin{align*}
S_{i+1} = U \Omega_{[\mu_i]} \Sigma Y_i V^T. \quad (17)
\end{align*}
$$

With $J_{i+1}$ and $S_{i+1}$ calculated, we show how to solve $Z_{i+1}$ and $L_{i+1}$. By dropping terms independent of $Z$, we obtain

$$
\begin{align*}
\tilde{\mathcal{A}}(Z,Y_i,Y_i,\mu) = \frac{\gamma}{2} \{Z^T L^{(0)} Z\} + \langle Y_i, X - XL - E \rangle + \langle Y_i, Y_i \rangle, \quad (18)
\end{align*}
$$

Through computing $\tilde{\mathcal{A}}(Z,Y_i,Y_i,\mu) / \partial \varepsilon = 0$, we can infer the solution of $L$ at the $(k+1)$-th iteration as
\[
Z_{i+1} = \left( I_n + \hat{\gamma} X^T X \right)^{1/2} \times \\
\left[ X^T (X - Y_i X - E_i) + J_{i+1} + \frac{1}{\lambda} \left( X^T Y_i^T - Y_i^T \right) \right] \times Z_{i+1} = \max( Z_{i+1}, 0 )
\]

where \( I_n \) is an identity matrix on \( \mathbb{R}^n \). Similarly by dropping terms independent of \( Z \) from \( \mathcal{S} \), we have

\[
\mathcal{S}(L, Y, Y, \mu) = \frac{1}{2} \text{Tr} \left( LX^T X E \right) + \{ Y_i, X - XZ - LX - E \} + \{ Y_i, L - S \} + \{ L - 3 \}. 
\]

By setting \( \mathcal{S}(L, Y, X, \mu) / \| L \|_0 = 0 \) and letting \( I_1 \) be an identity matrix on \( \mathbb{R}^* \), the solution of \( L \) at the \((k+1)\)-th iteration can be similarly inferred as

\[
L_{k+1} = \left[ (X - XZ_{k+1} - E_i) X^T + S_{k+1} + \frac{1}{\lambda} \left( Y_i^T X^T - Y_i^T \right) \right] \times L_k + \hat{\gamma}_k XZ^0 X^T + \hat{\gamma} X^T \]

D. Computing the Error Matrix \( E \)

After both \( Z_{i+1} \) and \( L_{i+1} \) are computed, the solution of sparse error matrix \( E_{i+1} \) can be inferred as

\[
E_{i+1} = \arg \min_{E} \| E \|_0 + \frac{1}{2} \| E \| - \left( \mathcal{S}(Y_i, E - \mathcal{S}^0) + \left( 1 / \lambda \right) Y_i^T \right). 
\]

Let \( \mathcal{S}^0 = X - XZ_{i+1} - L_{i+1} X \), we can achieve the sparse error matrix \( E \) at the \((k+1)\)-th iteration as

\[
E_{k+1} = \arg \min_{E} \| E \|_0 + \frac{1}{2} \| E - \mathcal{S}^0 \|_1 + \left( 1 / \lambda \right) Y_i^T. 
\]

Let \( \hat{\Theta}^0 = \hat{\Pi}^0 + \left( 1 / \lambda \right) Y_i^T \) according to \([12][11]\), denote by \( \hat{\Theta}^0 \), the \( i \)-th column of \( \hat{\Theta}^0 \), the \( i \)-th column \( E_{i+1} \) of solution \( E_{k+1} \) can be defined as

\[
E_{i+1} = \begin{cases} \\
\| \hat{\Theta}^0 \|_0 - \hat{\gamma} \left( \frac{1}{\lambda} \hat{\gamma} \right) & \text{if } \hat{\gamma} \left( \frac{1}{\lambda} \hat{\gamma} \right) < \| \hat{\Theta}^0 \|_0 \\
0 & \text{otherwise}
\end{cases}
\]

With all other variables obtained, the Lagrange multipliers \( Y_i \), \( \gamma \), and \( \lambda \) can be respectively undated by

\[
y_{i+1} = y_i + \mu_1 ( X - XZ_{i+1} - L_{i+1} X - E_{i+1} ), \]
\[
y_2 = y_2 + \mu_2 ( Z_{i+1} - J_{i+1} ), \]
\[
y_3 = y_3 + \mu_3 ( L_{i+1} - S_{i+1} ). 
\]

E. Construction of Weight Matrix \( W \)

The rLRR framework aims at preserving the similarity and locality among principal and salient features at each iteration. It is noted that data points connected within the high density regions on the data manifold are likely to have the same label \([19]\). Motivated by this assumption, rLRR is also proposed for encoding the relationship among the similar and local features. Let \( d_{ij} = \| X_i - X_j \| \) be Euclidian distance between points \( X_i \) and \( X_j \), when \( X_i \) is included in the \( k \) neighbors of \( X_j \), then rLRR assigns the weights \( W_{ij} \) using heat kernel \([19]\) as

\[
W_{ij} = \exp \left( -d_{ij}^2 / \sigma \right), \sigma = \sigma / \tau, \sigma = \sum_{i=1}^N d_{ij}^2 / (N^2 - N) \]
B. Exploratory Data Analysis

We first perform rLRR for exploratory data analysis. In this study, the MIT-CBCL face recognition database (Available at http://cbcl.mit.edu/software-datasets/FaceData2.html) is used. MIT-CBCL database includes images of 10 persons, and has two sets: (1) High-resolution images, including frontal, half-profile and profile view; (2) Synthetic images (324 images per person) rendered from 3D head models of ten persons. The images are captured under different illuminations, poses up to about 30 degrees of rotation. The second set is applied in this simulation and images are resized to 20×20 pixels. Figure 1 show typical samples and 30 faces per person (300 images totally) are selected. We first compare the adjacency matrix of the rLRR-graph with those of LatLRR-graph, LRR-graph and the sparse $l_1$-graph in Figure 2. For each algorithm, all model parameters are carefully chosen. Seeing from Figure 2, we can find that: (1) All the graphs implicitly emphasize the natural clusters within images of the same person. The weights of the adjacency matrices of rLRR-graph, LatLRR-graph and LRR-graph are denser than that of the $l_1$-graph. We also conclude that all of these graphs can deliver strong clustering structures and more discriminant information. We can also observe from the zooming in of rectangular regions denoting two persons that multimodal structures within each person can be clearly preserved by each method; (2) By explicitly emphasizing the similarity and locality among the close features, the weights of our rLRR-graph are denser with better groups, compared with the LatLRR-graph and LRR-graph.

We in Figure 3 illustrate the importance of emphasizing the similarity and locality preservation among features. Based on the results of Figure 2, we compare the right reconstruction coefficients $z_i^*$ and $z_j^*$ of two similar faces, $i$ and $j$, from the 3rd person. Note that these two faces are very similar in poses except for the degrees of illuminations, but the coefficients of LatLRR and LRR are greatly different, and more inter-person connections are produced. In contrast, the results exhibited by rLRR are promising, since the similarities among features are clearly kept by rLRR. The number of inter-person connections is also greatly reduced. More importantly, observing from the reconstruction coefficients of the faces of the 3rd person in the braces, we find the corresponding region of rLRR is densely clustered and most important values appear in this area.

C. Robust Representation of Natural Images

This experiment mainly evaluates rLRR for representing and decomposing natural images. Three natural image databases, i.e., butterflies, birds and textures, are used. The butterflies (7 classes), birds (6 classes) and textures database (25 classes) are collected by UIUC and are publicly available from http://www-cvr.ai.uiuc.edu/poncegrp/data/. In this study, the images of each database are resized to 32×32 pixels. Note that some original natural images to be decomposed and decomposable images are illustrated in Figure 4. Similarly, for a given image data matrix $X$, rLRR decomposes it into a low-rank part $Xz^*$ encoding principal features, a low-rank part representing the
salient features $LX$ and an error part $E^*$ fitting noise. From Figure 4, the principal features represent the noise removed principal information included in the original images and the salient features represent some key local parts of images, e.g., bodies of butterflies and birds, or textures of textures. That is, both principal and salient features are useful in representing and recognizing images. By properly preserving the similarity between the principal features and discriminative information included in salient features, rLRR will be promising for image representation and recognition.

$$X = XZ^* + L^*X + E^*$$

**Figure 4**: Natural image decomposable results of our rLRR on: (a) Butterflies, (b) Birds, (c) Textures.

D. Face Recognition

This study examines the projection matrix of LatLRR and our rLRR algorithm for feature extraction and face recognition. After the optimal projection matrix $L^*$ is achieved, feature extraction can be performed by projecting the coming test data onto $L^*$. The Euclidean one-nearest-neighbor (1NN) classifier is used. In this simulation, the second Synthetic set of MIT-CBCL is employed and we resize the images to 20×20 pixels due to the computational consideration. For face recognition, the dataset is randomly split into the training and test sets. The training set is applied to train the projection matrix of LatLRR and our proposed rLRR. Prior to feature extraction, PCA is used to remove the null space of the training set and the face samples are normalized to [0, 1]. In this study, we test LatLRR and rLRR by varying the training number per person from 14 to 22 with step 2. The experimental results are given in Table 1. We find that our rLRR delivers comparable and even better results the LatLRR algorithm in most cases.

<table>
<thead>
<tr>
<th>Method</th>
<th>MIT (14 train)</th>
<th>MIT (16 train)</th>
<th>MIT (18 train)</th>
<th>MIT (20 train)</th>
<th>MIT (22 train)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LatLRR</td>
<td>Mean 0.8800</td>
<td>Mean 0.9165</td>
<td>Mean 0.9433</td>
<td>Mean 0.9686</td>
<td>Mean 0.9783</td>
</tr>
<tr>
<td>rLRR</td>
<td>Mean 0.8926</td>
<td>Mean 0.9158</td>
<td>Mean 0.9441</td>
<td>Mean 0.9693</td>
<td>Mean 0.9802</td>
</tr>
</tbody>
</table>

V. CONCLUDING REMARKS

This paper has proposed a regularized low-rank representation (rLRR) framework for data representation and decomposition. The core idea is to enhance the robustness and performance of the recent LatLRR by incorporating a Laplacian regularization term to preserve the locality among those similar features in the iterative optimizations. As a result, the similarity and the desirable low-rank characteristics among principal and salient features can be effectively preserved by rLRR simultaneously. We mainly evaluate rLRR for exploratory data analysis, image representation and face recognition. The experimental results show that the presented rLRR delivers a strong capability in grouping features and preserving the similarity between them. Image decompositions also demonstrate the canonical features can be extracted by our rLRR formulation.

In future work, the following two directions can be further studied. First, it is observed from the simulations of the paper, the sparsest properties can be on a par with the lowest-rank characteristics for image representation and recognition. Thus
exploring whether one can further improve the performance of the proposed rLRR algorithm, for instance robustness, through incorporating the sparsest criterion is worth exploring. Second, extending rLRR for subspace learning [21][22] in addition to extract the features is another interesting future direction.

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