Adaptive linear learning for on-line harmonic identification: An overview with study cases

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Abstract—This work reviews Adaline-based techniques for estimating Fourier series. The Adaline, with its linear structure and learning, fits a Fourier series by expressing any periodic signal as a sum of harmonic terms. The learning with elementary harmonic inputs enforces the weights to converge to the amplitudes. The Adaline therefore individually identifies the amplitudes of the harmonic terms present in the measured signal in real-time. Relevant study cases are provided. Performances are evaluated and show that harmonic terms of the signals are efficiently estimated.

I. INTRODUCTION

Harmonic content is a fundamental concept in power system analysis, operation, and control; hence, its fast and precise estimation is of prime importance. Consequences and problems induced by higher-order harmonic terms in power systems have been well established [1]. Digital devices with high computational capabilities will expand the design of new and precise harmonics identification techniques.

Fourier-based approaches are among the most fundamental techniques in frequency analysis processing. However, they imply sliding window implementations and convolution operations which make their computational requirements a heavy burden in most applications. Furthermore, Fourier-based approaches only provide a response after a complete period of the measured signal and cannot calculate the dynamic characteristics of measured signals over time because of the assumption that analyzed signals are stationary [2]. Since the harmonic content varies constantly in power systems, fast and real-time estimation techniques are necessary for efficient actions.

An adaptive linear element, i.e., an Adaline, perfectly learns any linear weighted sums. Thus, if sine and cosine terms with multiple frequencies are its inputs and if a measured signal serves as a reference for its output, then the Adaline will learn a Fourier series. After convergence, the weight associated to each input, i.e., to a given harmonic term, represents its amplitude. A kind of LMS (Least Mean Square) algorithm updates its weights. This learning algorithm is not only extremely simple but also linear, which makes learning fast and easy. The amplitude of the harmonic terms can thus be iteratively updated, it is possible to access to them on-line in real-time applications.

The objective of this paper is to review the principles of using Adaline to estimate the amplitudes of Fourier series terms. The identification of periodic signals is illustrated by typical examples.

This paper is organized as follows. Section II briefly outlines some key issues of the Adaline network. Section III is devoted to the principle of learning Fourier series with an Adaline and variants are reviewed. Fourier series of typical signals are learned and their harmonic terms are individually extracted in Section IV. Section V concludes the paper.

II. BASICS OF ADAPTIVE LINEAR ELEMENTS

The architecture of the Adaline is based on a very simple unit which performs a processing. This unit consists of weights, a bias and a summation function, they are shown on Fig. 1. The processing comprise the calculation of the output for given inputs and the learning phase, i.e., the weights adjustment. The Adaline is able to fit any linear relationships by providing a scalar output as a weighted sum of the inputs and by adapting its weights. When a multidimensional output space must be considered, i.e., when several outputs are required, several Adaline having the same inputs are used and this is sometime referred to as a MADaline (Multiple Adaline).

Let \( x \) and \( w \) be two vectors, respectively for the inputs and the weights. For mathematical convenience, let the first element of \( x \) be equal to 1, so that the first element of \( w \) becomes the bias weight. At instant \( k \), the output \( \hat{y}(k) \) is a weighted sum given by the following dot product:

\[
\hat{y}(k) = w^T(k) x(k).
\]  

The Adaline network is a supervised learning network that needs to associate a reference value for each input vector. This reference is a desired value corresponding to an input and expressed in the Adaline’s output space. When an input \( x(k) \) is presented to the network, the output \( \hat{y}(k) \) is calculated and compared to the desired output \( y(k) \) that is associated to him. This defines the error

\[
e(k) = y(k) - \hat{y}(k) = y(k) - w^T(k) x(k).
\]  

The pairs of input/output values \( x(1), y(1), x(2), y(2), \ldots x(Q), y(Q) \) represents the learning data set. Each pair can be used on-line to adapt the weights at each iteration in order to minimize \( e(k) \). The new value of the
weight vector is updated from its previous value according to the μ-LMS rule or to the α-LMS rule, i.e., respectively

\[ w(k + 1) = w(k) + \mu e(k)x(k), \quad (3) \]

\[ w(k + 1) = w(k) + \alpha \frac{e(k)x(k)}{\|x(k)\|^2}, \quad (4) \]

where \( \mu \) and \( \alpha \) are learning rates. The α-LMS algorithm is only a normalized version of the μ-LMS algorithm. Normalizing the input \( x \) before applying it to the network leads to the same result using the μ-LMS algorithm. These learning rules come from the LMS algorithm and is called the Widrow-Hoff learning rule [3]. Approximately, the Adaline converges to least squares error when \( k \to \infty \) [4], [5]. The main characteristic of LMS algorithm is that it safes the error and it reduces the average quadratic error. Variants have been recapitulated in [6].

In (3) and (4), \( w(k + 1) \) is the new value that will take the weight vector from its previous value \( w(k) \) which represents the memory of the network. In the weight update process, the learning rate gives more or less importance to the innovation term based on the error compared to the memory term \( w(k) \). Therefore, the values of the rates \( \mu \) or \( \alpha \) are chosen between 0 and 1.

### III. REVIEW OF ADALINE-BASED APPROACHES TO HARMONIC CONTENT ESTIMATION

The last decades have seen many studies about harmonic distortion identification techniques to improve power quality. In this literature, a harmonic term is defined as a component of a periodic wave having a frequency that is an integral multiple of the fundamental power line frequency. In the following, we focus on on-line and iterative algorithms to estimate harmonic terms in real-time applications.

#### A. General case and principle of operation

Any periodic, distorted waveform can be expressed as a sum of pure sinusoids. The sum of sinusoids is referred to as a Fourier series. The Fourier analysis permits a periodic distorted waveform to be decomposed into an infinite series containing a DC component, a fundamental component (50/60 Hz for power systems) and its integer multiples called the harmonic components. The harmonic number \( n \) usually specifies a harmonic component, which is the ratio of its frequency to the fundamental frequency.

An ideal power signal, i.e., a voltage or a current, is a sinusoidal signal of period \( T \) (scalar)

\[ y(k) = a \sin(\omega k + \phi) \]

where \( a \) is the amplitude, \( \omega = 2\pi/T \) represents the actual angular frequency, and \( \phi \) is the initial phase angle. This signal is measured and digitalized with a sampling frequency of \( f_s \), the time interval between two successive samples is thus \( T_s = 1/f_s \).

A non-ideal power signal contains harmonic terms and noise and can generally be approximated by

\[ y(k) = \sin(\omega_0 k + \phi_0) + \sum_{n=2}^{N} a_n \sin(n\omega k + \phi_n) + \eta(k) \]

where \( a_0 \) is the DC component and \( \eta(k) \) represents a noise. Each harmonic component is defined by its amplitude \( a_n \) and its phase angle \( \phi_n \). Practically, the sum of the harmonic components \( a_n \sin(n\omega k + \phi_n) \) is limited to \( n = N \).

According to Fourier, every periodic signal can be estimated by a function \( f \):

\[ f(k) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega k) + \sum_{n=1}^{\infty} b_n \sin(n\omega k) \]

where \( a_0 \) is the DC part and \( n \) is called the \( n \)-th harmonic. The sum of the terms \( a_n \cos(n\omega k) \) is the even part and the sum of the terms \( b_n \sin(n\omega k) \) is the odd part of the signal. Rearranging the even and the odd part gives (8) which is a well known result:

\[ f(k) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega k - \theta_n), \quad (8) \]

with \( c_n \) the harmonic amplitudes and \( \theta_n \) the phase angles:

\[ c_0 = a_0, \quad c_n = \sqrt{a_n^2 + b_n^2}, \quad \theta_n = \tan^{-1} \left( \frac{b_n}{a_n} \right) \]

\( f(k) \) is a weighted sum of sinusoidal terms. It is therefore a linear relationship that can be fitted by an Adaline but with a limited number of \( n = N \) terms. Sine and cosine terms with unit amplitude are provided as the Adaline’s input vector:

\[ x = \begin{bmatrix} 1 & \sin(\omega k) & \cos(\omega k) & \ldots & \sin(N\omega k) & \cos(N\omega k) \end{bmatrix}^T \]

The output \( \hat{y}(k) \) of the Adaline is compared to the measured signal \( y(k) \) in order to adapt the weights. After learning and convergence, the signal is estimated by the function \( \hat{f}(k) = w^T x \) and we obtain:

\[ w^* = \begin{bmatrix} \frac{a_0}{2} & b_1 & a_2 & b_2 & \ldots & b_N & a_N \end{bmatrix}^T \]

This approach allows to handle the most general case, i.e., to estimate the amplitudes of the complete harmonic content of a signal with an Adaline. The Adaline can be designed to fit some precise harmonic content. Indeed, specific cosine and sine terms must be expressed, even with delays (i.e., \( \phi_n \)) and used as inputs. After learning and convergence, the Adaline fits the weighted sum and the weights represent the amplitude of each of them.
Generally, power system signals present a limited number of specific harmonics. Adalines with specific harmonic inputs is therefore very convenient for estimating the amplitude of current distortions.

B. Adaline-based approaches for harmonic estimation

In power systems, identifying the harmonics allows to separate the disturbing higher-order harmonics introduced by non-linear loads from the fundamental term carrying the electric energy [1]. These operations are necessary for monitoring and ensuring electric power quality. Efficient methodologies for the analysis and measurement of the basic electric magnitudes in are required. Methods with short computation time for real-time calculation must be employed for the generation of compensating currents in order to instantly re-inject them, most often with shunt active power filtering schemes [7].

The following shows how Adaline-based approaches can be judiciously used for estimating the frequency/harmonic content of power system signals. Frequency estimation means estimating the fundamental frequency and tracking its fluctuations and deviations. Harmonics identification means estimating the amplitude and the phase of the harmonic terms contained in the signal.

The use of an Adaline to learn the Fourier series of the signal model given by (6) has been introduced in [8]. This work corresponds to the general approach detailed above where a decaying DC quantity is added to the signal model. An additional element \((kT_s)\) is therefore introduced in the Adaline input vector and allows to efficiently track the amplitude and the phase of 6 harmonic terms. A similar approach is proposed in [9], where a signal model with a different expression of the decaying quantity is used. This leads to the modification of one element of the input vector. The estimation error is also fed back recurrently in order to enhance the input vector by 3 elements \((e(k), e(k−1) \text{ and } e(k−2))\). The very simplest approach, based on the Fourier series, is also used in [10] and in [11]. In this last work, only the two weights elements of the fundamental component are updated, hence it is independent of the number of harmonic orders present. In [12], the same approach and the same signal model is used, one Adaline is used for harmonic estimation, another Adaline is used for predicting the line voltage.

The S-Adaline proposed in [13] is able to synchronize itself with time-varying signals to the frequency deviation for online tracking of single phase reactive power. It contains a fundamental angular frequency deviation measurement algorithm that is used to generate sine and cosine terms of the input vector of the Adaline. These terms are thus in phase with the fundamental term of the measured signal. In [2], two Adalines in a cascaded two-stage approach is used. In the first stage, an Adaline implements the Prony’s method for tracking the fundamental frequency of the measured signal. In the second stage, an Adaline learns the Fourier series decomposition of the signal with the very simplest approach for estimating the amplitudes of the harmonics.

The previous approaches identify the harmonics in the measured signal reference frame. This means that the measured signal, i.e., the current, is directly expanded into a Fourier series which is learned by an Adaline. However, the measured signal can be converted into another reference frame before being expanded, learned and approximated by an Adaline. If the principle remains the same, the conversion of the signal in a different reference frame allows to highlight more or less some parts of the signal. The current is thus converted into a virtual power space by multiplying the measured current by a sine term in [14]. In another method of [14], 2 Adalines serves to estimate the Fourier series of the instantaneous PQ-powers [7] which requires the measure of the currents and of the voltages for the 3 phases. In [6], measured current of the 3 phases is converted into a current expressed in the DQ-space with the Park transform. A Complex Adaline is proposed in [15]. This approach estimates the fundamental frequency of a power system with an input vector composed of sine and cosine terms. To produce the input vectors and deal with the decaying DC term, the Park transformation is used. The two weights associated to the fundamental frequency are used through a Hamming filter to calculate the amplitude of the fundamental term.

The Adaline for frequency estimation and harmonic identification can be used in a different way by replacing the Fourier series expression by a recursive linear expression of the signal. Considering a measured signal of the type given by (5), three consecutive samples \(y(k), y(k−1)\) and \(y(k−2)\) meet the relationship

\[
y(k) = (2 \cos \omega_0 T_s) y(k−1) - y(k−2)
\]  

The inputs of the Adaline therefore become \(x = [ y(k−1) \; y(k−2) ]^T\) and its output is compared to the reference signal \(y(k)\) with is the measured signal at instant \(k\). After minimizing the error, the weights converge to \(w^* = [ w_0^* \; w_2^* ]^T = [ 2 \cos \omega_0 T_s \; -1 ]^T\). The frequency can thus be obtained from the first element of \(w^*\). Indeed, \(\hat{f} = (2 \pi T_s)^{-1} \cos^{-1}(w_0^*)\). As can be seen, it is simple and therefore well suited for the frequency estimation problem.

However, it is sensitive to noise because based on an ideal expression of a current waveform.

This simple principle is used in [16] where the Adaline inputs are enhanced by additional harmonic terms to take account of a decaying DC component and of harmonic distortions present in the power system signal. Fundamental frequency estimation is thus achieved. In [17], a tapped delay line of the measured current is used to generate the inputs for the Adaline. Power quality event detection is thus possible with an Adaline with only 4 inputs. In [18], a tapped delay line of a larger size is used to generate the Adaline inputs for disturbance detection. The identification of the power system frequency is achieved by another Adaline that combines delayed signal measures and sine and cosine inputs. More recently, [19] proposes an approach for frequency estimation but not with an Adaline. It is a least-squares approach that uses 3 consecutive measures.
of the signal and that calculates once per iteration the solution (i.e., the coefficients equivalent to the weights of the Adaline) by using a pseudo-inverse computing.

Finally, we can say that:

- There are two main approaches, one based on learning and approximating the amplitudes of the Fourier series used to express the signal, and one based on learning and approximating an iterative expression of the signal.
- Learning the Fourier series is more appropriate to harmonic terms identification while learning the iterative expression is more appropriate to fundamental frequency estimation.
- Approximating Fourier series can be tackled in the measured signal’s but also in other specific reference frames.
- All the reviewed approaches use a sophisticated learning rule with an adaptive learning rate.

IV. STUDY CASES

All these approaches are well suited for power system applications where typical nonlinear loads generate very low even harmonics and where triple harmonics can be ignored in three-phase circuits [1]. As a consequence, appropriate input terms can be specified for the Adaline. Once identified, the harmonic terms can be compensated individually according to the strategy objective (full or selective harmonic compensation, power factor and unbalance compensation...). It is obvious that the dimension of the weight vector to be updated on each iteration depends on the number of harmonics to be estimated. The complexity of the implementation must be compliant to real-time constraints.

In order to show the effectiveness of the approach, different cases are investigated thereafter. At the beginning, the Adaline is initially untrained with \( \mathbf{w}(0) = 0 \). For convenient reasons, the learning rate is chosen as constant: \( \alpha = 0.85 \) for all the studied cases. Finally, the MSE (Mean Square Error) of the estimation is used as a measure of overall performance.

A. A main sinusoid with harmonics of ranks 3, 5 and 7

As a first example, we propose to identify the harmonic content of a signal composed of a fundamental sinusoidal term where harmonics of rank 3, 5 and 7 have been added as follows:

\[
\begin{align*}
    s(k) &= \sin \omega k + \frac{1}{3} \sin 3\omega k + \frac{1}{5} \sin 5\omega k + \frac{1}{7} \sin 7\omega k \\
    &= \sum_{n=1}^{20} \alpha_n \sin n\omega k 
\end{align*}
\]  

(13)

with a fundamental frequency \( f = 50 \) Hz. The signal is sampled with \( T_s = 0.0002 \) s. and 4000 samples are used.

If only the frequency of the fundamental term is known and if we suppose that the signal does only contain harmonic terms of rank less than 20, then its harmonic content can be estimate by an Adaline that fits the classical Fourier series given by (7) with 20 terms. This means that the Adaline will take 41 inputs, 1 DC term, 20 cosine and 20 sine terms. The values of the coefficients \( a_0, a_n, b_n \) will be estimated by learning.

After convergence, the amplitudes are obtained from \( \mathbf{w}^* \): \( a_0 = 0.0, a_{n=1,2,\ldots,20} = \{0.0\} \) and \( b_{n=1,2,\ldots,20} = \{1.0, 0.0, 0.3333, 0.0, 0.2000, 0.0, 0.1429, 0.0, \cdots, 0.0\} \).

These coefficients perfectly represent the harmonic content of \( s(k) \) in (13). Results obtained with an Adaline with 41 inputs are represented by Fig. 2. The error between the original signal \( s(k) \) and the one estimated by the Adaline is represented. Additionally, the signal and its estimation, but also the evolution of the Adaline’s weights are shown. The weights that converge to zero correspond to the harmonic components that are not present in the signal. It can be seen that after 3 periods of signal, the MSE is less than 0.1 \( (s(k) \) varies between -1 and 1). Furthermore, the MSE is less than \( 8e^{-11} \) over the two last periods.

On the other hand, if we know which harmonics are present in the signal for estimating their amplitude, then we precisely specify them as the inputs of the Adaline. In the case of \( s(k) \) given by (13), knowing that it is composed of a DC component, of a the fundamental term with \( f = 50 \) Hz and of harmonics of rank 3, 5 and 7 allows to defined the following inputs:

\[
\mathbf{x}(k) = \begin{bmatrix}
    1 & \sin \omega k & \sin 3\omega k & \sin 5\omega k & \sin 7\omega k
\end{bmatrix}^T .
\]

(14)

After learning, the resulting weights are:

\[
\mathbf{w}^* = \begin{bmatrix}
    0 & 1.0 & 0.3333 & 0.2000 & 0.1429
\end{bmatrix}^T .
\]

(15)
One can see that the Adaline affects a null weight to the DC component and correctly estimates the amplitude of the harmonic terms (fundamental and higher-order harmonics). The estimated signal, the error and the weights evolution are represented by Fig. 3. The MSE is less than 0.04 after 3 periods and is less than 3e-011 over the two last periods. With only 5 inputs, the degrees of freedom of the Adaline are less than with 41 inputs and the convergence is therefore faster.

The computational complexity and computing time are not the same in both cases. Intensive simulations tests show that there is a ratio of approximately 1/12 between the time to learn with 5 and with 41 inputs under the same computing conditions.

Similar tests have been lead to signals with a fundamental term and up to 30 randomly chosen higher-order harmonics. An Adaline with 30 sine and cosine terms is able to estimate these signals with an final MSE of less than 5e-5 (for signals varying between -1.5 and 1.5). Knowing the present harmonics allows to precisely specify the inputs in order to obtain their amplitude and allows to reduce the computational complexity and a faster convergence. Obviously, delayed harmonic terms can be taken into account. As an example, $\frac{1}{7}\sin(7\omega_k)$ has been replaced by $\frac{1}{7}\sin((7\omega_k - \pi)/4)$ in (13). The results show that with the appropriate inputs, the Adaline is perfectly able to estimate the signal with the same precision.

**B. Case of a pure square wave**

The example of a periodic square wave that varies between 0 and 1 and that rises for the first time at $k = 0$ is interesting because in theory it is estimated by the following series:

$$f(k) = \frac{1}{2} + \frac{2}{\pi} \left( \sin \omega_k + \frac{1}{3} \sin 3\omega_k + \frac{1}{5} \sin 5\omega_k + \cdots \right)$$

which gives in other words, $a_0 = 1/2$, $a_n = 0$, $\forall n$, and $b_{n=1,2,3,4,5,6,\cdots} = \{0.6366, 0.0, 0.2122, 0.0, 0.1273, 0.0, 0.0909, 0.0, \ldots\}$.

A square wave signal with $f = 40$ Hz and composed of 8000 samples has been artificially synthesized with $T_s = 0.0002$ s. Assuming that such a signal is made up to $N = 20$ harmonic terms, an Adaline with 41 weights can estimate it. After learning with an Adaline configured like this, we obtain from $w^*$: $a_0 = 0.4996$, and $a_{n=1,2,3,\cdots,20} = \{-0.0089, 0.0021, -0.0664, 0.0204, -0.0052, -0.0033, 0.0012, \cdots\}$ and $b_{n=1,2,3,\cdots,20} = \{0.6253, 0.0091, 0.2111, 0.0062, 0.1271, 0.0040, 0.0888, \cdots\}$. Furthermore, all amplitudes of harmonic terms of rank over 7 are less than 0.0533. It can be noticed that the values of $a_n$ and $b_n$ when $n$ takes an even value are close to zero. Thus, the estimated values of the amplitudes perfectly corresponds to the theoretical Fourier series of (16). Results are illustrated by Fig. 4 and the MSE between the perfect square signal and the output of the Adaline, i.e., the recovered signal, is 0.0641 for the 2 last periods.

**C. Case of real noisy signals**

1) A real noisy square wave signal: The example of a non perfect square wave that varies between -0.3 and 0.3 which $f = 40$ Hz is chosen. It is a real signal, disturbed by noise, and composed of 80 000 samples measured with a sampling period of $T_s = 0.0002$ s.

As in the previous cases, an Adaline with the same parameters and under the same conditions is used to estimate its conventional Fourier series. Results with 41 weights ($N = 20$) are given by Fig. 5 and the MSE is 0.0396 for the 2 last periods of the signal. It can be noticed that the noise still remains on the recovered signal. It has not been removed because it has not been modeled in one of the weights as a harmonic term.

2) A current measured on a nonlinear low power load: The harmonic content of a typical current of a nonlinear load power device is estimated thereafter. This current is measured on one of the 3 phases of a nonlinear load characterized by $f=50$ Hz, 380 V, 20 A and 4000 samples are used ($T_s = 6.0000e-005$ s.).

This signal is analyzed with an Adaline with the same parameters and under the same conditions. For a Fourier series with $N = 20$, an Adaline with 41 weights leads to a MSE of 0.1931 over the last two periods. In a second test, a Fourier series with $N = 60$ is estimated by an Adaline with 121 weights. Results are represented by Fig. 6 where the harmonic terms are also as a histogram. They are estimated in less than 3 periods. The MSE over the last two periods is 0.1093 which represents an error of 0.45% for the current (on a range of 24 A).

3) An Electrocardiogram: The electrocardiogram (ECG) is a graphical representation of the electrical activity of the
real power system currents and square waves have been efficiently estimated. Tests with other types of signals, such as ECG, demonstrate the good performance of the Adaline in estimating harmonics. Whatever the type of the signal, the harmonics are identified with a MSE of less than 0.45% and in less than 3 periods of the signal.

REFERENCES