Neuro-Fuzzy Control Strategy for Methane Production in an Anaerobic Process

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Abstract—In this paper, a neuro-fuzzy control strategy composed by a neural observer and fuzzy supervisors for an anaerobic digestion process is proposed in order to maximize methane production. A nonlinear discrete-time recurrent high order neural observer (RHONO) is used to estimate biomass concentration and substrate degradation in a continuous stirred tank reactor. A Takagi-Sugeno supervisor controller based on the estimation of biomass, selects and applies the most adequate control action, allowing a smooth switching between open loop and closed loop. The control law calculates dilution rate and bicarbonate rate based on speed-gradient inverse optimal neural control. Finally, Takagi-Sugeno supervisors calculate reference trajectories for the system states, and gain scheduling for the dilution rate control law at different operating points of the process. The applicability of the proposed scheme is illustrated via simulations.

I. INTRODUCTION

Anaerobic digestion (AD) processes are very attractive because of their waste treatment properties and their capacity for generating methane from waste materials, which can be used for electrical energy generation [1]-[4]. AD is a biological process by which organic matter (substrate) is degraded by anaerobic bacteria (biomass), in absence of oxygen. Biogas production through anaerobic degradation is primarily composed of methane (CH$_4$) and carbon dioxide (CO$_2$). AD is a complex and sequential process which takes place in four basic stages: hydrolysis, acidogenesis, acetogenesis and methanogenesis [5]. Each stage has a specific dynamics; hydrolysis, acidogenesis and acetogenesis are fast stages in comparison with methanogenesis, which is the slowest one; it imposes the dynamics of the process and is considered as the limiting stage [6].

On this paper, the AD process is developed in a continuously stirred tank reactor (CSTR), with immobilized biomass on a solid support, and limiting stage is modeled in order to produce as much biogas as possible and a good quality soil amendment. The substrate pH is an important parameter for the adequate growth of bacteria and then for the wastes transformation. Dilution rate ($D_{in,k}$) and bicarbonate supplying rate ($b_{inc,k}$) are used in order to regulate pH of the process. A discrete-time nonlinear mathematical model of the process is developed in [5]. The physical-chemical phenomena (acid-base equilibria and material conservation) are modeled by algebraic equations and the biological phenomena are modeled by ordinary differential equations, which represent the dynamical part of the process.

A variety of factors affect the rate of digestion and biogas production, such as pH, temperature and overloads [7]. In addition, there exist variables and parameters hard to measure due to economical or technical constraints. Then, estimation and control strategies are required in order to guarantee adequate performance [8].

Methane production, biomass growth and substrate degradation are good indicators of the biological activity inside the reactor. These variables can be used for monitoring the process and to design control strategies. Sensors have been developed in order to measure methane production in bioprocesses [9], [10]. However, substrate and biomass measures are more restrictive. The existing biomass sensors are quite expensive, and are designed from the biological viewpoint (based on capacitance or turbidity properties); hence, they are not reliable for control purposes. Furthermore, substrate measure is done off-line by chemical analysis, which requires at least two hours. State observers are an interesting alternative in order to deal with this situation.

A discrete-time recurrent high order neural observer (RHONO) for unknown nonlinear systems in presence of external disturbances and parameter uncertainties [11], [12] is proposed in order to estimate biomass concentration and substrate degradation in the anaerobic process. This neural observer is based on a discrete-time recurrent high order neural network [13] (RHONN) trained with an extended Kalman filter (EKF)-based algorithm, using a parallel configuration. The training of the RHONO is performed online. The variables are estimated from CH$_4$ and CO$_2$ flow rates, which are commonly measured in this process.

In order to control the anaerobic digestion model, a speed-gradient inverse optimal neural control for trajectory tracking [14] is considered; which is developed on the basis of the above mentioned neural observer. The controllers determine two control actions, bicarbonate supplying rate ($b_{inc,k}$) and dilution rate ($D_{in,k}$), in order to track a methane reference trajectory production. A Takagi-Sugeno (TS) supervisor [15], [16] detects activity of the process based on estimated biomass, selects and applies the most adequate control action, allowing a smooth switching between open loop and closed loop. The process works in open loop in presence of small disturbances. A TS supervisory detects the disturbance amplitude on input substrate and implements a fuzzy interpolation to obtain the
nonlinear reference trajectories for systems states and CH₄ production. Finally, fuzzy gain scheduling for the dilution rate controller is implemented. This neuro-fuzzy control strategy improves the performances of the anaerobic process at different operation points and is feasible for application in real processes.

II. DISCRETE TIME RHONO

A discrete-time RHONO [17] structure which estimates the variables of the methanogenesis stage: biomass (X₂,k, mol/L), substrate (S₂,k, mol/L) and inorganic carbon (IC₀,k, mol/L) is presented. The biomass is used in the TS supervisor controller structure, and the substrate and the inorganic carbon are estimated for comparison (with real measures) in future researches. The observability property of this AD process is analyzed in [18]. It is concluded that substrates (S₂,k and S₂,k), biomasses (X₂,k and X₂,k) and inorganic carbon (IC₀) are observable states. On Fig. 1, the proposed observer scheme is displayed and its structure is shown on equations (1).

\begin{align}
\dot{X}_{2,k+1} &= w_{1,k}^2X_{2,k} + w_{1,k}S_{2,k} + w_{1,k}^1\hat{IC}_0 \\
&+ w_{1,k}^1S_{2,k}D_{m,a} + w_{2,k}^1S_{2,k}S_{m,a,k} + g_{m,a}\varepsilon_1, \\
\dot{S}_{2,k+1} &= w_{2,k}^1\hat{S}_{2,k} + w_{2,k}S_{2,k} \hat{S}_{2,k} + w_{2,k}S_{2,k}\hat{IC}_0 \\
&+ w_{2,k}S_{2,k}D_{m,a} + w_{2,k}S_{2,k}S_{m,a,k} + g_{m,a}\varepsilon_1, \\
\dot{\hat{IC}}_{0,k+1} &= w_{0,k}^1\hat{IC}_0 + w_{0,k}S_{2,k}\hat{IC}_0 + w_{0,k}S_{2,k}X_{2,k} \\
&+ w_{0,k}S_{2,k}\hat{IC}_0D_{m,a} + w_{0,k}S_{2,k}\hat{IC}_0S_{m,a,k} + g_{m,a}\varepsilon_1, \\
&\text{where } k \in \mathbb{Z}^+ = \{0, 1, 2, \ldots\}, w_0 \text{ is the respective on-line adapted weight vector; } X_{2,} S_{2,} \text{ and } IC \text{ are the estimated states; } S(\cdot) \text{ is the sigmoid function defined as } S(x) = \alpha \tanh(\beta x); D_m (1/h) \text{ is the input dilution rate, } S_{2m,k} \text{ (mol/L) is the input substrate, } b_{m,k} \text{ (mol/L) the bicarbonate addition and } e \text{ is the output error.}
\end{align}

The gaseous phase CH₄ (mol/h) and CO₂ (mol/h) is considered as the process output:

\begin{align}
\hat{Y}_{CH_4} &= R_{1,k} X_{2,k} \\
\hat{Y}_{CO_2} &= R_{2,k} X_{2,k},
\end{align}

where μ₂,k the growth rate (Haldane type) of X₂,k (1/h), R₁, R₂, R₃ are the yield coefficients and λₖ is a pressure partial coefficient for CO₂ defined as:

\begin{align}
\lambda_t &= \frac{CO_{2d}}{P_{K_a}-CO_{2d}},
\end{align}

where CO₂d (mol/L) is the dissolved carbon dioxide P, is atmospheric pressure (atm) and Kₐ is a gases Henry constant (mol/L atm).

A. EKF based algorithm

EKF based algorithms have been introduced to train neural network (NN), improving learning convergence. Since the NN mapping is nonlinear, an EKF-type is required. The training goal is to find the optimal weight values, which minimize the predictions error. More details are presented in [19], [20]. The used EKF-based training algorithm is described by (4).

\begin{align}
w_{i,k+1} &= w_{i,k} + \eta_i K_{i,k} e_{i,k}, \\
K_{i,k} &= P_{i,k} H_{i,k} M_{i,k}, \\
P_{i,k+1} &= P_{i,k} - K_{i,k} H_{i,k} P_{i,k} + Q_{i,k}, \quad i = 1, \ldots, n,
\end{align}

with

\begin{align}
M_{i,k} &= \left[ R_{i,k} + H_{i,k}^T P_{i,k} H_{i,k} \right]^{-1}, \\
e_{i,k} &= y_k - \hat{y}_k,
\end{align}

where e_i,k ∈ R^p is the observation error, P_{i,k} ∈ R^{l×l} is the prediction error covariance matrix at step k, w_{i,k} ∈ R^l is the weight (state) vector, L_i is the respective number of neural network weights, y ∈ R^p is the plant output, \hat{y} ∈ R^p is the NN output, η_i is the learning rate, K_{i,k} ∈ R^{l×p} is the Kalman gain matrix, Q_{i,k} ∈ R^{l×l} is the NN weight estimation noise covariance matrix, R_{i,k} ∈ R^{p×p} is the error noise covariance, and H_{i,k} ∈ R^{p×p} is the matrix for which each entry (H_{i,j}) is the derivative of the i-th neural output with respect to j-th NN weight, (w_{ij}). Where i = 1, n and j = 1, L_i. Usually P_{i,0} and R_{i,0} are initialized as diagonal matrices, with entries P_{0,0}, Q_{0,0} and R_{0,0} respectively. It is important to remark that H_{i,k}, K_{i,k} and P_{i,k} for the EKF are bounded [21].

B. Tuning Guidelines

It is assumed that the weights values are initialized as small random values with zero mean and normal distribution.

Fig. 1 Observer scheme
The learning rate \( (\eta) \) determines the magnitude of the correction term applied to each neuron weight; it is usually bounded to \( 0 < \eta < 1 \). Thus, if \( \eta \) is small then the transient estimated state is over-damped; if \( \eta \) is large then the transient estimated state is under-damped; finally if \( \eta \) is larger than a critical value then the estimated state is unstable. Therefore, it is better to set \( \eta \) to a small value and to increase it if necessary. More details are discussed in [19].

The Luenberger-like observer gain \((g_1, g_2, g_3)\) is set by trial and error; unfortunately there is a shortage of clear scientific rationale to define it. However, it is adequate to select \(0 < g < 0.1\) for a good performance on the basis of training experience.

### III. SPEED-GRADIENT INVERSE OPTIMAL NEURAL CONTROL

The speed-gradient inverse optimal neural control is based on the above described neural observer. The aim of the inverse optimal neural control consist in design a stabilizing feedback control law, based on an a priori known control Lyapunov function (CLF) which ensure that the stabilizing control law optimizes a cost functional [22]. Let consider the affine in the input discrete-time nonlinear system:

\[
x_{k+1} = f(x_k) + g(x_k)\ u_k
\]

where \(x_k \in \mathbb{R}^n\) is the state of the system, \(u_k \in \mathbb{R}^m\) is the control input, \(f(x_k) : \mathbb{R}^n \to \mathbb{R}^n\) and \(g(x_k) : \mathbb{R}^n \to \mathbb{R}^{m} \) are smooth maps, \(k \in \mathbb{Z}^+ = \{0, 1, 2, \ldots\}\). \(f(0) = 0\) and rank \(\{g(x)\}_{x \in \mathbb{X}} = m \forall x_k \neq 0\).

Reference [23] establishes the following cost functional associated with system (5):

\[
V(z_k) = \sum_{n=1}^{\infty} \{ l(z_n) + u_n^T R_c(z_n) u_n \}
\]

where \(z_k = x_k - x_{d,k}\) with \(x_{d,k}\) as the desired trajectory for \(x_k; z_k \in \mathbb{R}^n\); \(V(z_k) : \mathbb{R}^n \to \mathbb{R}^+\); \(l(z_k) : \mathbb{R}^n \to \mathbb{R}^+\); \(R_c(z_k) : \mathbb{R}^n \to \mathbb{R}^{m \times m}\) is a positive semidefinite positive definite weighting matrix. The cost functional (6) is a performance measure [24]. The entries of \(R_c(z_k)\) can be functions of the system state in order to vary the weighting on control efforts according to the state value [24]. Considering state feedback control, we assume that the full state \(x_k\) is available. Equation (6) can be rewritten as

\[
V(z_k) = l(z_k) + u_k^T R_c(z_k) u_k + \sum_{n=1}^{\infty} \{ l(z_n) + u_n^T R_c(z_n) u_n \}
\]

where we require the boundary condition \(V(0) = 0\) so that \(V(z_k)\) becomes a Lyapunov function. In order to establish the conditions that the optimal control law must satisfy, we define the discrete-time Hamiltonian \(H(z_k, u_k)\) [25] as

\[
H(z_k, u_k) = l(z_k) + u_k^T R_c(z_k) u_k + V(z_{k+1}) - V(z_k).
\]

A necessary condition that the optimal control law \(u_k\) should satisfy is \(\partial H/\partial u_k = 0\), [24] which is equivalent to calculate the gradient of (8) right-hand side with respect to \(u_k\), then

\[
0 = 2R_c(z_k) u_k + \frac{\partial V(z_{k+1})}{\partial u_k}
\]

\[
= 2R_c(z_k) u_k + \frac{\partial z_{k+1}}{\partial u_k} \frac{\partial V(z_{k+1})}{\partial z_{k+1}}
\]

\[
= 2R_c(z_k) u_k + g^T (x_k) \frac{\partial V(z_{k+1})}{\partial z_{k+1}}
\]

Then, we propose a CLF, [23] \(V(z_k)\), such that control law will be inverse optimal (globally) stabilizing along the desired trajectory \(x_{d,k}\). Hence, instead of solving (8) for \(V(z_k)\), a quadratic candidate CLF \(V(z_k)\) for (11) is proposed with the form:

\[
V_c(z_k) = \frac{1}{2} z_k^T P_k z_k, \quad P_k = P^T_k > 0
\]

Consequently, by considering \(V(z_k)\), control law (11) takes the following form:

\[
u_k^* = -\frac{1}{2} \left[ R_c(z_k) + \frac{1}{2} g^T (x_k) P_k g(x_k) \right]^{-1} g^T (x_k) x_k^* \]

\[
P_k \left( f(x_k) - x_{d,k+1} \right).
\]

It is worth to point out that \(P_k\) and \(R_c(z_k)\) are positive definite and symmetric matrices; thus, the existence of the inverse in (13) is ensured. To calculate \(P_k\), which ensures trajectory tracking of \(x_k\) for system (5) with (13) along the desired trajectory \(x_{d,k}\), we use the speed-gradient (SG) algorithm. Control law (13) at every time step depends on the matrix \(P_k\). Let define this matrix as:

\[
P_k = P_c P^T C
\]

where \(P_c = P_c^T > 0\) is a given constant matrix and \(p_k\) is a scalar parameter to be adjusted by the SG algorithm [14]. Then, \(13\) is transformed into:

\[
u_k^* = -\frac{P_c}{2} \left[ R_c(z_k) + \frac{1}{2} g^T (x_k) P_k g(x_k) \right]^{-1} \times
\]

\[
g^T (x_k) P_k \left( f(x_k) - x_{d,k+1} \right).
\]

The SG algorithm is now reformulated for the trajectory tracking inverse optimal control problem. The dynamic variation of parameter \(p_k\) on (15) results in

\[
p_{k+1} = p_k + 8 \gamma_{d,k} \times
\]

\[
\left( f(x_k) - x_{d,k+1} \right)^T \left[ P_k g(x_k) R_c(z_k) g^T (x_k) f(x_k) - x_{d,k+1} \right] \left( 2R_c(z_k) + p_k g^T (x_k) P_k g(x_k) \right)
\]

\[
\left[ 2R_c(z_k) + p_k g^T (x_k) P_k g(x_k) \right]
\]
which is positive for all time step \( k \) if \( p_0 > 0 \). Therefore, positiveness for \( p_k \) is ensured and requirement \( P_k = P_k > 0 \) for (14) is guaranteed. With the CLF as defined by (12) and \( p_k = \pi \) (\( \pi \) is a constant value when the SG algorithm converges), the control law is inverse optimal in the sense that it minimizes the cost functional (6), which is proposed such that \( l(x) \) ponders the states and \( R_C \) ponders the control [14].

According to (15), the inverse optimal control law for AD is formulated as

\[
u'_i - D_{in,k} = -\frac{1}{2} \left( R_C (\hat{x}_i) + \frac{1}{2} g_i^T (\hat{x}_i) P_{in} g_i (\hat{x}_i) \right) \times \\
g_i^T (\hat{x}_i) P_{in} g_i (\hat{x}_i) f (\hat{x}_i, x_{\theta,k+1})
\]

\[
u'_i - b_{inc,k} = -\frac{1}{2} \left( R_C (\hat{x}_i) + \frac{1}{2} g_i^T (\hat{x}_i) P_{inc} g_i (\hat{x}_i) \right) \times \\
g_i^T (\hat{x}_i) P_{inc} g_i (\hat{x}_i) f (\hat{x}_i, x_{\theta,k+1})
\]

(17)

where the positive definite matrix \( P_k = p_k P_C \) is calculated by the SG algorithm, \( R_C (\hat{x}_i) \) a constant matrix and \( f (\hat{x}_i, x_{\theta,k+1}) = f (\hat{x}_i) = x_{\theta,k+1} \). The tracking of a desired trajectory is based on the separation principle for discrete-time nonlinear system [23, 26].

IV. FUZZY SUPERVISOR STRUCTURE

A. Fuzzy TS supervisor controller

A TS supervisor [15, 16, 27] for the controller has two main tasks: i) detect the process state, and ii) select the most adequate control action allowing smooth switching (if required) between them. The idea is to detect the attraction region where the process is working; if any operating conditions cause the process to move away from the operating domain, the supervisor must determine and apply the control action which allows the biomass to grow in order to avoid washout. Besides, if a variation on the operation conditions can be managed by the process itself, the supervisor must allow the system to operate in open loop, which represents energy saving. Organic daily load per biomass unit (ODL/X) variable is important regarding process stability [6], and is proposed for the fuzzy inference rules. ODL/X represents the maximal quantity of organic load that a biomass unit can treat during a working day. The input disturbances can be classified by this variable into small, average and large. For this reason, three fuzzy sets are determined as shown in Fig. 2.

The fuzzy set limits are obtained doing independently tests of each one of the control actions. Concerning the output fuzzy variables, three operation regions for the process are identified: open loop, closed loop with \( b_{inc} \) action, and closed loop with \( D_m \) action.

The TS algorithm [15] is used to define the supervisor. From empirical knowledge, each fuzzy set is associated with a control action; then three fuzzy inference rules are deduced (18-20):

If \( ODL/X \) is LOW then \( u_i = \) open loop \hfill (18)

If \( ODL/X \) is AVERAGE then \( u_i = b_{inc,k} \) action \hfill (19)

If \( ODL/X \) is HIGH then \( u_i = D_{in,k} \) action \hfill (20)

Fuzzy values can be transformed in real values using the centre-average defuzzification. This method is selected because is an interpolator between the consequents, which are defined by the premise of fuzzy rules [15]:

\[
u = \frac{\sum_{j=1}^{R} \gamma_j \{ u_j \}}{\sum_{j=1}^{R} \gamma_j}
\]

(21)

with \( R \) the number of rules, \( \gamma_j \) is known as the membership function and is calculated as \( \gamma_j = \gamma_{ODL/X} \), where \( \gamma_{ODL/X} \) is the membership degree of variable \( ODL/X \) on the fuzzy set, \( k \) the \( k \)th fuzzy set of \( ODL/X \) and \( \sum_{j=1}^{R} \gamma_j = 1 \).

The \( ODL/X \) is defined as:

\[
ODL/X = D_{in,k} A_2 S_{20} / \hat{X}_2
\]

(22)

where \( D_{in,k} \) is the dilution rate (1/h), \( A_2 \) a disturbance amplitude on the substrate input \( S_{20} \) (mol/L), \( S_{20} \) the initial value of the substrate \( S_2 \) and \( \hat{X}_2 \) is the estimated biomass \( X_2 \) (mol/L). The structure of the TS supervisor controller is shown on the figure 3.

B. Fuzzy TS supervisor for reference trajectories

The inverse optimal neural control algorithm requires reference trajectories to force the system to track them. The proposed reference trajectories are taken from a previous work [28] because they represent an optimal dynamic

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Fig. 2 ODL/X fuzzyfication

Fig. 3 TS supervisor control scheme
behavior of the anaerobic process for growth biomass and methane production in presence of disturbances. Five trajectories from low to high disturbance on input substrate are calculated. In order to obtain global reference trajectories in presence of different disturbances, TS supervisor for AD reference trajectories are developed. The inference rules are composed of linguistic variables as premises and reference trajectories varying-time (instead of linguistic variables) as consequents [29]. For the premises, the input disturbance $S_{2in}$ is selected as the fuzzy input variable and 5 fuzzy sets are proposed, as illustrated in Fig. 4.

![Fig. 4 $S_{2in}$ fuzzyification](image)

Each fuzzy set corresponds to different amplitude of the disturbance; for each amplitude, a reference trajectory is synthesized. Five inference rules are deduced as follows:

- If $S_{2in}$ is VERY LOW then $x_{ref} = x_{r1}$, $y=C_1x_{ref}$ (23)
- If $S_{2in}$ is LOW then $x_{ref} = x_{r2}$, $y=C_2x_{ref}$ (24)
- If $S_{2in}$ is AVERAGE then $x_{ref} = x_{r3}$, $y=C_3x_{ref}$ (25)
- If $S_{2in}$ is HIGH then $x_{ref} = x_{r4}$, $y=C_4x_{ref}$ (26)
- If $S_{2in}$ is VERY HIGH then $x_{ref} = x_{r5}$, $y=C_5x_{ref}$ (27)

$x_{ref} = x_{ik}$ corresponds to reference trajectories for $X_k$, $S_2$ and $IC$. From this fuzzy rules structure, it is easy to see that the active reference trajectories at each instant are determined by $S_{2in}$. The global reference trajectory is calculated using the defuzzification [29] algorithm described by:

$$ x_{ref} = \frac{\sum_{i=1}^{5}\gamma_j \cdot x_{r_i}}{\sum_{j=1}^{5}\gamma_j} $$  \hspace{1cm} (28)

$$ YCH_{x_{ref}} = \frac{\sum_{j=1}^{5}\gamma_j \cdot C_jx_{ref}}{\sum_{j=1}^{5}\gamma_j} $$  \hspace{1cm} (29)

where $\gamma_j$ is the membership function and determined as $\gamma_j = \gamma_j^{S_{2in}}$ with $\gamma_j^{S_{2in}}$ the membership degree of variable $S_{2in}$ on the respective fuzzy set.

The scheme of the supervisory reference trajectories is inserted and shown on the whole neuro-fuzzy control scheme (Fig. 5).

C. Fuzzy TS supervisor gain scheduling

With this new approach, the control algorithm requires to tune the gains of the $D_{in}$ controller. In order to simplify this tuning task, a fuzzy gain scheduling is implemented [29].

For the rules, the input disturbance $S_{2in}$ is selected as the input variable. A TS supervisor detects the disturbance amplitude on the input substrate and implements a fuzzy interpolation to keep updating the gain of the controller.

Fuzzy sets are the same as illustrated in Fig. 4.

For each one of the disturbance amplitude, an inverse optimal neural control (17) is synthesized in order to regulate the substrate around an operating point. The corresponding gains of the nonlinear controller are determined according to the inverse optimal control approach and are used as output variables in the consequents. Hence, the fuzzy rules have the structure:

- If $S_{2in}$ is VERY LOW then $K_f = K_{D1}$ (30)
- If $S_{2in}$ is LOW then $K_f = K_{D2}$ (31)
- If $S_{2in}$ is AVERAGE then $K_f = K_{D3}$ (32)
- If $S_{2in}$ is HIGH then $K_f = K_{D4}$ (33)
- If $S_{2in}$ is VERY HIGH then $K_f = K_{D5}$ (34)

The global gain is calculated using the defuzzification equation described by:

$$ K_f = \frac{\sum_{j=1}^{5}\gamma_j \cdot K_{Dj}}{\sum_{j=1}^{5}\gamma_j} $$  \hspace{1cm} (35)

where $\gamma_j$ is the membership function and determined as $\gamma_j = \gamma_j^{S_{2in}}$, with $\gamma_j^{S_{2in}}$ the membership degree of variable $S_{2in}$ on the respective fuzzy set. The scheme is inserted and shown on Fig. 5.

![Fig. 5 Neuro-fuzzy control scheme](image)
V. SIMULATION RESULTS

The whole control strategy for the anaerobic process is implemented using Matlab.\textsuperscript{TM} The observer is initialized at random values to verify the estimation convergence. In order to test the observer sensitivity to input changes, a disturbance of 50\% $S_{Z_{2m}}$ increase on the input substrate is incepted at $t = 200$ hours. The performance of the proposed RHONO is illustrated on Fig. 8.

![Fig. 8 State estimation for a 50\% disturbance](image1)

System states are well estimated. Thus, the proposed neural observer is a good alternative to estimate those important states of the considered anaerobic process. Model and observer validation are found in [28]. The neuro-fuzzy control strategy performance is tested considering simulations close to experimental conditions. The presented results correspond to the experiment with the largest experimentation time.

First, the proposed control strategy is tested in presence of a 110\% disturbance on $S_{Z_{2m}}$, incepted at $t=200$ hours. Trajectories tracking for states and $YCH_4$ are illustrated in Fig. 9.

![Fig. 9 State estimation for a 110\% disturbance](image2)

On this test, the process operates in open loop because disturbance is small; the $ODL/X_2$ belongs to the associated fuzzy set corresponding to open loop (Fig. 2). This situation implies that the AD process is able to work adequately without control (constants $b_{inc,k}$ and $D_{in,k}$) in presence of this small disturbance. Thus the response of the system is stable during the process time.

The control strategy scheme is tested introducing a large disturbance of 200\% $S_{Z_{2m}}$ incepted at $t=200$ hours, as illustrated on Fig. 10.

\textsuperscript{TM}: The Mathworks Inc., Cambridge, MA, USA
When the disturbance is introduced on the input substrate, the proposed scheme detects a large disturbance and the $ODL/X_2$ belongs to the associated fuzzy set corresponding to closed loop (Fig. 2). The fuzzy supervisors determine adequate reference trajectories and control law in order to allow biomass growth, diminishing $ODL/X_2$ and stabilizing the system.

As is illustrated on Fig. 10, trajectory tracking for the states and $YCH_4$ are efficient and the error approaches zero on the steady state. System operating is ensured due to the control strategy applied, even though a large disturbance is incepted.

Finally, tolerance to change on the system parameters is tested; such variation is incepted as a disturbance on the bacteria concentration, $\mu_{max}$ and $\mu_2^{max}$ and the same disturbance as in previous test incepted at t=200 h. Performance of the system is illustrated on Fig. 11.

As is illustrated on Fig. 11, the closed loop performance presents a transient state error due to parameters variation on rate growths which affect directly the kinetic; the control scheme acts in order to reject the disturbances diminishing $ODL/X_2$ and stabilizing the system. The reference trajectory is achieved on steady state with a small error. Biomass is well estimated and methane production reference trajectory
is tracking with small error. Thus, the robustness of the proposed RHONO to parameters variations is verified.

VI. CONCLUSIONS
A neuro-fuzzy control strategy for an anaerobic wastewater treatment process is proposed in order to produce methane and to avoid washout. A nonlinear discrete-time recurrent high order neural observer (RHONO) is used to estimate the biomass concentration and substrate degradation. Once this model is obtained, an inverse control law, based on it, is developed. The TS supervisor controller detects biological activity inside the tank reactor, on the basis of estimated biomass, and applies the adequate control action. The TS supervisor for reference trajectory detects disturbance amplitude on the input substrate, and global reference trajectories are determined. Finally, the TS supervisor gain scheduling interpolates adequate gain for the dilution action controller on the basis of disturbance amplitude on the input substrate. The goal is to force the system to track desired references; simulation results show how the neuro-fuzzy control scheme is able to stabilize the methane production along desired trajectories in presence of disturbances, and avoiding washout. This research will be pursued in order to evaluate the application on real-time of the proposed scheme to an anaerobic prototype process.

ACKNOWLEDGMENT
This work is supported by CONACyT projects 131678 and 105844.

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